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Cortical bases of elementary deductive reasoning: Inference, memory, and metadeduction

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ABSTRACT

Elementary deduction is the ability of unreflectively drawing conclusions from explicit or implicit premises, on the basis of their logical forms. This ability is involved in many aspects of human cognition and interactions. To date, limited evidence exists on its cortical bases. We propose a model of elementary deduction in which logical inferences, memory, and meta-logical control are separable subcomponents. We explore deficits in patients with left, medial and right frontal lesions, by both studying patients' deductive abilities and providing measures of their meta-logical sensitivity for proof difficulty. We show that lesions to left lateral and medial frontal cortex impair abilities at solving elementary deductive problems, but not so lesions to right frontal cortex. Furthermore, we show that memory deficits differentially affect patients according to the locus of the lesion. Left lateral patients with working memory deficits had defective deductive abilities, but not so left lateral patients with spared working memory. In contrast, in medial patients both deductive and meta-deductive abilities were affected regardless of the presence of memory deficits. Overall, the results are compatible with a componential view of elementary deduction, and call for the elaboration of more fine-grained models of deductive abilities.

1. Introduction

Being able to grasp the deductive relations among sentences or thoughts is a fundamental cognitive ability. If you want to go to a movie and your friend says that if it rains she will not come, and then if it does rain, you will not wait for her. Successful exchanges of information among people, or planning of novel action sequences, require the ability to carry out such deductive inferences: our everyday mental life is densely populated by them.

Deductive reasoning is often much more complex. It is involved in mathematics, formal logic, categorization, and scientific hypothesis testing and confirmation. Yet, while most people will never engage in sophisticated logico-mathematical reasoning in their life, the kind of everyday reasoning we exemplified above is arguably universal. As basic deductive steps are also involved in word learning (Halberda, 2003), elementary reasoning is also likely to appear early in development. By contrast, most mathematical or sophisticated logical reasoning requires years of training and appears late in life. Different levels of performance also support the contrast between the two types of deductive abilities. While humans solve simple deductive problems involved in everyday reasoning almost flawlessly (e.g. Braine, Reiser, & Rumain, 1984), once they go beyond this level of elementary reasoning errors abound. The relations between early basic reasoning abilities and the more sophisticated ones, such as explicit logico-deductive or mathematical reasoning, are unclear. However, what is apparent is that deductive reasoning is a multi-faced phenomenon, not necessarily involving only a single psychological mechanism. In this article, we will concentrate on the basis of elementary reasoning abilities, that is, the deductive abilities that every human being possesses and deploys in everyday exchanges of information.

An elementary level of deductive inference is presupposed by both main theories on human deduction – mental models and mental logic – along with more sophisticated reasoning abilities. According to mental logic theory, reasoning involves the construction of short mental proofs, built by means of a set of rules and procedures for their application. According to one of the most developed version of this theory (e.g., Braine & O'Brien, 1998a), reasoners possess rules in natural deduction form that govern the introduction or elimination of connectives and quantifiers. A procedure for the application of those rules (called Direct Reasoning Routine,

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hereafter *DRR*) allows reasoners to move from premises to conclusions in a finite number of steps. The set of natural deduction rules and its associated DRR are meant to describe the universal elementary reasoning skills. In contrast, complex abilities responsible for individual differences depend on the acquisition of secondary reasoning strategies. These are not primitive, do not appear early in development, and not everybody develops them. Critically, it is assumed that they require the involvement of cognitive processes qualitatively different from the ones involved in elementary reasoning. Evidence consistent with the mental logic theory of elementary reasoning has been found in abstract problem solving (Braine et al., 1995), proof understanding, text understanding, memory for stories and lexical retrieval (Lea, 1995; Lea et al., 1990), almost exclusively within the domain of propositional reasoning.

In the mental model framework, a clear distinction between primary and secondary reasoning is not so explicit, but it is still present. The mental model theory holds that reasoning consists in the construction of analogical structures that mirror real states of affairs (e.g., Johnson-Laird, 1983, p. 419; Johnson-Laird & Byrne, 1991). In contrast with the mental logic theory, such structures do not require rule-like logical operations, or the explicit representation of variables or quantifiers: the ability to represent examples of real states of affairs grounds reasoning. Besides reasoning proper, the mental model theory also aims at explaining understanding and text comprehension (Johnson-Laird, 1983; Garnham, 1987). According to the theory, understanding a conversation, or a text, or a set of premises, means to build a first model consistent with it (Johnson-Laird & Byrne, 1991). This first step of comprehension is spontaneous and automatic. By contrast, the creation of further alternative models consistent with the premises or text (which are necessary to undertake more sophisticated reasoning) requires effort, motivation, and is severely limited by memory resource allocations. Thus, within the mental model theory, the difference between first comprehension and construction of alternative models grounds the difference between an intuitive level of model construction involving the creation of one or two models, and a non-automatic, more taxing level of model construction.

The aim of the present article is to advance the understanding of this intuitive and elementary level of reasoning presupposed by both theories.

1.1. *Elementary deduction, metadeduction and the brain*

Although psychological theories tend to present deduction as an all-or none process even at its elementary stage deduction is a complex phenomenon with several aspects. One prominent aspect is the deduction sequence itself, that is, the passage from one step to another during the search for a conclusion from a set of premises. A second aspect is meta-deductive: it consists in the ability to keep track of the unfolding of a deductive reasoning, by locating how single steps relate to the overall structure of a reasoning process. A third aspect is the ability to temporarily store the representations needed to either produce a deduction sequence or to supervise the structure of a deduction sequence. Potentially, deficits to any one of the three components may produce reasoning impairments.

Because the main aim of a theory of reasoning is to predict how subjects will respond to deductive problems, naturally most reasoning research has focused on the deduction process rather than on the other components. For deduction, mental logic holds that the complexity of a (elementary) problem is a function of the length of the proof needed to evaluate or generate the conclusion weighted by the individual difficulties of the rules entering the proof. By contrast, for the mental model theory the difficulty of a problem is a function of the number of models required to validate a putative conclusion. To explore the predictions of both theories, the percentage of correct solutions to deductive problems is a rough but fundamental index. Several studies show that length of proof is a good predictor of participants' errors (Braine, 1998; Braine et al., 1984, 1995; Braine & O'Brien, 1998a; Lea, 1995; Lea et al., 1990; Yang, Braine, & O'Brien, 1998). Other studies suggest that also the number of models correlates with errors (Johnson-Laird & Bara, 1984; Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999; Johnson-Laird, Byrne, & Schaeken, 1992; Schaeken, Johnson-Laird, & d'Ydewalle, 1996). Such studies assume that models can be unambiguously counted, which may not be the case (Bonatti, 1994, 1998).

Sensitivity to proof structure, although fundamental has received less attention, and almost exclusively within the framework of the mental logic theory. This interest is easily understood. A deductive proof is a structured object, often requiring several intermediate subgoals. To reach a conclusion, a reasoner must apply individual rules, but also maintain a general sense of their position within the reasoning process, and/or a conception of the distance between the premises and the conclusion, over and above each single step involved in the proof. By monitoring the structure of the reasoning process, s/he can better judge the distance between current state and the final goal, and thus allocate resources that are needed to solve the problem. A failure to do this may lead to errors. For example, in solving a problem a reasoner may be led to explore the consequences of a supposition, perhaps before finally discarding it. However, if s/he does not monitor the overall reasoning process, s/he may think s/he has reached a final conclusion about the consequences while in fact s/he is still within an intermediate step of proof construction, one in which the current intermediate conclusion is valid only under that supposition. This insensitivity of the structure of a proof will lead the reasoner to make errors. Evidence exists that participants are sensitive to the suppositional structure of a mental proof (Marcus & Rips, 1979).

Sensitivity to overall problem structure has been mostly studied by probing the difficulty judgments of the participants for elementary deductive problems. Studies have shown that participants can form stable and well differentiated difficulty judgments, even for elementary reasoning problems (e.g., Braine et al., 1984), and that they correlate with different degrees of proof complexity.[1]

In complementary fashion, the role of memory in deduction has been studied especially within the mental model frameworks. Again, the reason for this interest can be easily understood. As models are memory structures, a natural prediction of mental models is that limited working memory will reduce reasoning abilities. This prediction has been explored with varying degree of success by studying developmental differences in reasoning abilities, or by using tasks varying concurrent memory load (Copeland & Radvansky, 2004; De Neys, Schaeken, & d'Ydewalle, 2005a, 2005b; Markovits, Doyon, & Simoneau, 2002).

In short, all three dimensions of elementary deductive reasoning have been recognized and studied in the psychological literature, although not within the unified frame which we propose. Clearly, an approach that could assess the respective role of these factors would be highly informative for theories of deduction. For this purpose, studies on the involvement of the frontal cortex in elementary reasoning can be extremely useful.

We have argued that, in order to carry out elementary deductions, first, rules for deriving inferences (or procedures to construct

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[1] Also mental models can be stretched to correlate with perceived difficulty, but at the price of severely reducing the coherence of the overall explanation of elementary deduction. Because elementary reasoning requires at most two models, the model theory is not fine-grained enough to capture differences of perceived difficulty within elementary reasoning. Thus, the mental model theory could show a relation between models and difficulty judgments only at the cost of changing how models for elementary reasoning are counted (Bonatti, 1994). If one allows for this change in counting criteria, then mental models also correlate quite accurately with judgments of perceived difficulty (Johnson-Laird et al., 1992).

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models) must be recruited; then, the ability to represent the overall unfolding of the proof is required, and finally, memory is needed in order to represent the premises, the intermediate states of a derivation, and its overall structure. In all these functions, the frontal cortex may play a crucial role. Several lines of evidence suggest that the subprocesses we postulate in elementary reasoning involve specific frontal regions. Firstly, in general, the frontal cortex seems to be involved during the execution of deductive tasks, more specifically the left lateral and medial cortex (Fangmeier, Knauff, Ruff, & Sloutsky, 2006; Goel, Buchel, Frith, & Dolan, 2000; Goel & Dolan, 2003; Monti, Osherson, Martinez, & Parsons, 2007; Reverberi et al., submitted for publication; Reverberi et al., 2007; but see also Reverberi, Rusconi, Paulesu, & Cherubini, 2009). Secondly the fronto-polar cortex (mainly, Brodmann Area 10) has been linked to "cognitive branching" or "multiple sub-goal scheduling", i.e. the human ability to hold in mind goals while exploring and processing secondary goals (Braver & Bongiolatti, 2002; Burgess, Veitch, de Lacy Costello, & Shallice, 2000; Koechlin, Basso, Pietrini, Panzer, & Grafman, 1999; Ramnani & Owen, 2004). As we recalled, this ability is important in order to build and manipulate the overall logical structure of a deductive problem. Finally, the dorsolateral frontal cortex is known to be involved in verbal working memory (Baddeley, 2003; D'Esposito & Postle, 1999; Owen, McMillan, Laird, & Bullmore, 2005). These abilities are all also required to carry out elementary deductions.

However, limited neuropsychological evidence is available about the role of frontal cortex in deduction. In particular, to our knowledge, its role in elementary deduction has not been directly corroborated by any neuropsychological study. Two neuropsychological group studies are available to date specifically aimed at investigating the role of the frontal cortex in propositional deductive reasoning (Goel, Shuren, Sheesley, & Grafman, 2004; Adolphs, Tranel, Bechara, Damasio, & Damasio, 1996). Such studies are unlikely to shed light on elementary reasoning, as they explore the neural basis of the Wason selection task (Wason, 1968). Although prominent in the psychological literature about reasoning, it is now widely agreed that despite its deceptively simple propositional-like form the Wason selection task taps onto several different inferential mechanisms (Girotto, Kemmelmeier, Sperber, & Jean-Baptiste, 2001; Sperber, Cara, & Girotto, 1995; Cosmides, 1989). As a consequence, it becomes very difficult to interpret neuropsychological results based on the Wason selection task.

The aim of our study is to provide novel evidence specifically addressing the role of frontal cortex in elementary propositional deduction and, if possible, to clarify the neurological basis of the functional distinction between the different dimensions of the deduction process we discussed.

2. Materials and methods

2.1. Participants

Thirty-six Italian patients with a single focal brain lesion as determined by a CT or an MRI scan were recruited from the Neurological and Neurosurgical wards of the Ospedale Civile in Udine (Italy). All patients gave their consent to participate to the study, which was approved by the ethical committee of SISSA-ISAS (Scuola Internazionale Superiore di Studi Avanzati - International School for Advanced Studies). The etiology of the patient sample was mixed: stroke, neoplasm and arachnoid cyst (Table 1). Exclusion criteria were the presence in the clinical history of psychiatric disorders, substance abuse or previous neurological disease, neuroradiological evidence of diffuse brain damage, diagnosis of language comprehension problems, and age less than 18 or more than 70. The time since the lesion ranged between 7 and 1579 days (Table 2); the starting date considered in the case of neoplasm is the day of surgery. Two patients had been diagnosed as mild Broca aphasics with no comprehension deficits. Twenty-five healthy control volunteers also participated in the study. Their age and educational level matched that of patients. A different Control Group ($n = 27$) was used as reference for the digit span forward and digit span backward tests (Table 2).

Table 1
Aetiology for each lesion group.

	MED	LL	RL	Overall
Arachnoid cyst	1			1
Glioma high grade	1	1	2	4
Glioma low grade	6	2	1	9
Meningioma	9	6	5	20
Stroke	1	2		3

MED: medial frontal; LL: left lateral frontal; RL: right lateral frontal.

2.2. Neuroradiological assessment

For all patients, a CT or an MRI scan was available. Following the general procedure of Stuss et al. (1998), patients were assigned to three anatomically defined subgroups according to their lesion site (Fig. 1). In the medial region group (MED), the lesion primarily involved the orbital surface and/or the medial surface of one or both frontal lobes (in the present study we did not split the medial group into superior and inferior groups because too few patients had a lesion to the inferior medial frontal lobe). The left lateral (LL) and right lateral (RL) patients had unilateral damage to the frontal lobe, primarily involving the lateral surface. In order to classify lesions, a senior neuroradiologist, blind to the experimental results, evaluated the scans. All patient lesions were mapped using the free MRIcro (www.mricro.com) software distribution (Rorden & Brett, 2000). Lesions were drawn manually by a senior neuroradiologist blind to the experimental results on slices of a T1-weighted template MRI scan from the Montreal Neurological Institute (www.bic.mni.mcgill.ca/cgi/icbm_view). This template is approximately oriented to match Talairach space (Talairach & Tournoux, 1988) and is distributed with MRIcro.

2.3. Materials and procedure

2.3.1. Deduction test

2.3.1.1. Stimuli. We created 30 stories in Italian, embodying different logical structures taken from a subset of the abstract problems used by Braine et al. (1984). All problems required only a few deductive DRR steps according to Mental Logic, or at most two models to be constructed according to Mental Models theory. As we wanted problems for which the expected performance in normal participants was fairly well known, we adapted stories studied behaviorally by Bonatti and Viti (in preparation) using Italian participants. Examples of the stories we proposed to our participants are in Appendix A.

Each story began with one sentence introducing the actors of the story and providing thematic focus (Lea et al., 1990; Lea, 1995). After the introductory sentence, every problem was composed of a variable number of premises (from 1 to 3), and a conclusion. Care was taken to reduce possible atmosphere effects by generating conclusions whose content was neutral with respect to the premises, so as to maximize resort to logical reasoning in order to judge the validity of the conclusion. Story length ranged from 24 to 68 words (mean word length = 45). Although the problems differed in word length, the stories were worded so as to minimize the possibility that simple problem length could account for possible difficulties in problem resolution.

In each story, the validity of the conclusion from the premises could be determined by applying rules of the Direct Reasoning Routine (Braine & O'Brien, 1998b), thus requiring only elementary reasoning. Half of the deductive problems had a valid conclusion, and the other half an invalid conclusion. The estimated difficulty of the problems we used ranged from 1.5 to 5.1 (on a scale from 1 to 9) as assessed by Braine et al. (1984) where the error rates ranged from 0 to 13%. Such estimates, obtained by testing English participants with abstract problem structures, corre-

Table 2
Demographic variables in each lesion group and in healthy control participants.

	MED	LL	RL	Patients overall	CTL	CTL-WM
N	18	10	8	36	25	27
Age [mean (SD)]	48 (11)	47 (13)	45 (14)	47 (12)	46 (9)	48 (10)
Education [mean (SD)]	10.2 (2.9)	10.3 (2.4)	10.5 (3.5)	10.3 (2.8)	10.6 (3.2)	9.6 (3.3)
Days from onset [median (range)]	360 (14–1507)	353 (7–1119)	375 (7–1230)	360 (7–1507)		
Lesion size (cc) [mean (SD)]	57.3 (47.3)	45.61 (39.0)	43.9 (25.9)	50.9 (40.4)		

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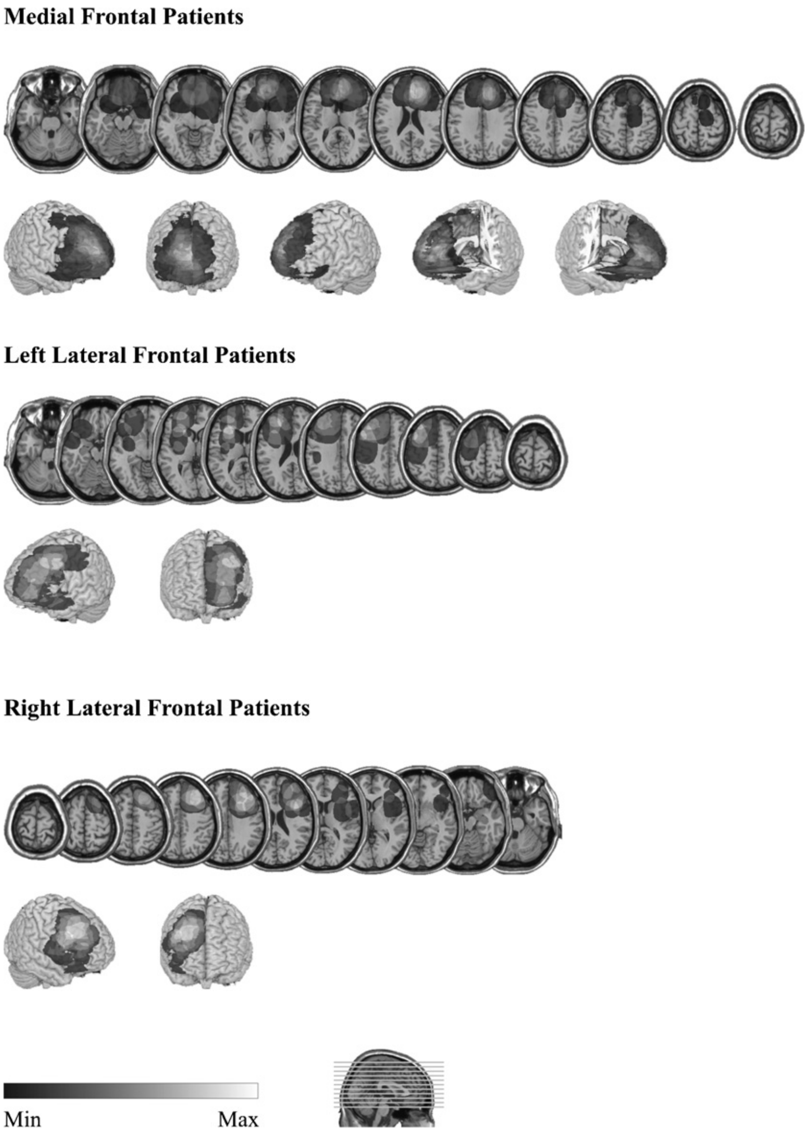

Fig. 1. Overlay lesion plots for the three lesion subgroups. The number of overlapping lesions in each voxel is illustrated on a grey scale: the lighter is a point on the plot, the higher the number of patients with that voxel damaged. The grey scale is devised so that the white color codes for the maximum overlap in each lesion subgroup; e.g. in the left lateral group the maximum number of patients with a lesion to the same voxel across the whole brain is six, thus the white color in the grey scale for LL group will code for six patients having the lesion (maximum overlap for RL is 6, for MED is 11).

spond fairly well to the estimates obtained by testing Italian participants with both abstract structures and contentful stories embodying those structures (Bonatti & Viti, in preparation). Comparison between patients and their matched Italian controls also provides further internal validation of the scaling data and the material in general.

2.3.1.2. Procedure. The experiment was carried out in Italian under the control of a personal computer running the E-Prime$^{\text{TM}}$ software (http://www.pstnet.com/products/e-prime/) and a button box.

At the beginning of the experiment, written instructions were displayed on the screen. Participants were informed that they would be asked to perform two basic tasks. First, they had to solve 30 simple deductive problems. Second, after having solved each of them, they had to rate their difficulty on a scale from 1 (very simple) to 9 (very difficult). To avoid training effects, pre-training was kept as light as possible, and participants saw only two example problems meant to fix the extreme points of the scale. Bonatti and Viti (in preparation) found that this procedure can elicit sufficiently different and stable difficulty judgments. Participants were informed that each premise problem had to be taken as true, regardless of its content, and they were only required to ask if the conclusion would follow from the given premises or not.

Each test problem started with a complete presentation of its premises (Fig. 2). After reading the premises, participants pressed a button to visualize the proposed conclusion and from that moment they had to judge its validity as fast and as accurately as possible. To avoid excessive memory load, during the presentation of the conclusion the premises remained on screen. After participants gave their response by pressing a button, the whole problem disappeared and they were asked to judge its difficulty. In order to help participants to assess the relative difficulty of each problem, a visual analogue of the difficulty scale was displayed on screen. As soon as participants verbally answered, the next trial started. No phase of the experiment was time-constrained.

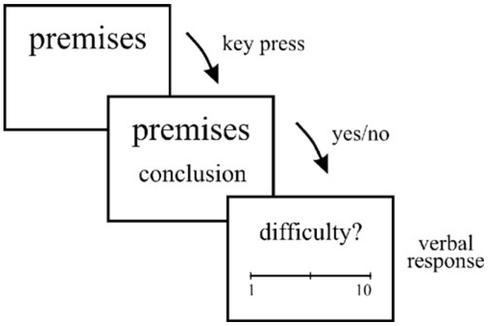

Fig. 2. Schema of stimuli presentation in a trial.

2.3.2. *Working memory assessment*

We assessed the ability of both control participants and patients to temporarily store verbal material by means of two standard short-term memory tests: the digit span forward and the digit span backward (Wechsler, 1997). Span was scored as the highest number of digits that a participant could correctly recall.

2.3.3. *Verbal comprehension*

We used the two most difficult subtests of the Token test, from the Italian version of the Aachener Aphasie Test (Luzzatti, Willmes, De Bleser, & Bianchi, 1994) to assess the ability to understand sentences. The test was administered only to the patients. For five patients in the Medial and Right Lateral subgroups these data are not available.

2.4. *Statistical analysis*

2.4.1. *Dependent variable of interest*

For the deduction test, we analyzed the following scores:

(i) *Accuracy*: the proportion of correct responses;
(ii) *Subjective difficulty*: the average difficulty ratings for each problem;
(iii) *Deviation from reference* for the subjective difficulty. To assess a reference score for the difficulty of each problem, we used the ratings of the Control Group. In order to obtain this index, first we translated each individual set of 30 ratings to the same individual mean (5) both for patients and healthy participants. Then, for each of the 30 problems we computed a reference value by averaging the translated ratings of the control group. From this value, we computed the square of the difference between each translated rating and its appropriate (same problem) reference rating. Finally, for each subject we calculated the individual average of the squared difference[2]; and
(iv) *Difficulty-accuracy consistency*, or, (for each problem) the consistency between the judged subjective difficulty and the ability to correctly solve it. For each participant, we classified each problem as being subjectively “difficult” if the difficulty rating was above the individual average over all problems, or “simple” if it was not. Then, for each participant we computed the probability that a problem was classified as “difficult” given that it had not been solved correctly.

While the two first measures are commonly used in behavioral studies on reasoning with normal participants, the third and fourth measures are specific to our study. The deviation from reference is meant to assess whether the patients' pattern of judgments differs from controls, thus bypassing the need of having a fully stabilized reference scale. A higher deviation from reference index with respect to controls can indicate an abnormality in meta-cognitive abilities. The difficulty-accuracy consistency is meant to assess to what extent patient judgments of subjective difficulty for a problem are predictive of their ability to solve it, always with respect to the same index in the control group. A lower level of prediction indicate abnormal meta-cognitive abilities. These measures indicate how patient meta-cognitive abilities (specifically, the ability to monitor the overall structure of proofs) diverge from the ability to carry out each individual step in proof construction. It is possible that the two abilities may dissociate: a patient could retain an ability to survey the general structure of a proof correctly, but be unable to carry out its steps while building it, or, vice-versa, be able to apply each rule step by step, but lose track of the overall structure of the proof. In these two cases, we would expect to find higher values of difficulty-accuracy consistency with respect to controls.

2.4.2. *Group analysis*

We first checked the raw data for conformity to the normal distribution with a Kolmogorov–Smirnov test, and for homogeneity of variance with a Levene Test. Variables differing significantly from the normal distribution or having inhomogeneous variances between groups underwent logarithmic transformation. If one of the assumptions of Analysis of the Covariance (ANCOVA) was still not valid after transformation, we analyzed the data with a non-parametric test (Mann–Whitney), estimating *p*-values with the exact method. Where an ANCOVA was carried out, the effects on the dependent variables were evaluated by covarying for age and education. Given our expectation on the direction of the effects for most of variables considered, we generally used one-tailed tests if not otherwise specified. Effects were considered significant at the $p < 0.05$ level.

3. Results

3.1. *Effect of demographic factors, time since lesion, lesion size and etiology*

We evaluated the effect of demographic factors, time since lesion and etiology on the accuracy score of the deduction test. Both age and education significantly affected the proportion of correct responses in the patient group (age: $R^2 = 0.232$, $F(1,34) = 10.274$, $p = 0.003$; education: $R^2 = 0.232$, $F(1,34) = 10.270$, $p = 0.003$). None of these factors were significant in the control group. We also ran a regression analysis with the logarithm of the days from onset of the disease (Table 2) as the independent variable, age and years of education as covariates and accuracy as a dependent variable. The proportion of the variance explained by days from onset was negligible ($R^2 = 0.018$, $F(1,32) = 0.872$, $p > 0.1$), as well as the variance explained by the lesion size ($R^2 < 0.001$, $F(1,31) = 0.019$, $p > 0.1$). Finally, we evaluated the possible effects of differences in etiology by performing an ANCOVA with demographic factors as covariates. Apart from arachnoid cyst ($n = 1$, not included) four groups were identified: meningioma ($n = 19$), high grade glioma ($n = 4$), low grade glioma ($n = 9$) and stroke ($n = 3$). Differences in etiology did not affect the deduction accuracy score ($F(3,29) = 1.391$, $p > 0.1$).

3.2. *Deduction test: accuracy*

Overall, frontal patients were less accurate than controls in the deduction test ($F(1,57) = 11.561$, $p = 0.0006$, Table 3). However, not all patient subgroups behaved in the same way. Both left lateral and medial patients were impaired ($F(1,31) = 7.376$, $p = 0.005$, and $F(1,39) = 10.726$, $p = 0.001$, respectively). By contrast, right lateral patients did not differ from controls ($F(1,29) = 1.495$, $p > 0.1$).

Because average lesion size was larger in the medial and left lateral group (see Table 2), we ran another ANOVA by excluding the patients with the largest lesions from the left lateral and medial subgroup. With such selection criteria, the average lesion size was similar in the three lesion subgroups (left lateral $n = 9$, lesion size $= 36.5\,\text{cm}^3$; medial $n = 14$, lesion size $40.7\,\text{cm}^3$). Even when lesion size was controlled, performance in the left lateral and medial subgroups was worse than in control subjects ($F(1,30) = 10.094$, $p = 0.002$, and $F(1,35) = 8.964$, $p = 0.003$, respectively). This result seems to exclude the possibility that lesion size by itself is responsible for the selective deficit of our patient subgroups.

3.3. *Deduction test: difficulty ratings*

Patients were not reliably different from controls in difficulty ratings ($F(1,57) = 0.459$, $p > 0.1$), nor was any patient subgroup. Even the deviation from reference index revealed no difference between patients and controls ($F(1,57) = 2.765$, $p > 0.1$). However, the subgroup analysis showed a selective difference for medial patients. In this subgroup, the index of deviation from reference was significantly higher than that of controls ($F(1,39) = 4.447$, $p = 0.021$). Neither the left lateral ($F(1,31) = 0.470$, $p > 0.1$) nor the right lateral subgroups ($F(1,29) = 0.017$, $p > 0.1$) showed such a difference.

Difficulty-accuracy consistency produced a similar pattern. Overall, patients did not differ from controls ($F(1,53) = 2.064$, $p > 0.1$). However, the subgroup analysis revealed that medial frontal patients deviated significantly from controls, unlike the other subgroups ($F(1,35) = 3.090$, $p = 0.043$ for medial patients; $F(1,27) = 0.304$, $p > 0.1$ for left lateral patients, and $F(1,25) = 0.136$, $p > 0.1$ for right lateral patients).

The difficulty-accuracy consistency measure expresses the conditional probability of a problem being rated as above-average difficulty given that it was incorrectly evaluated. As such, it depends

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[2] More precisely, the procedure to obtain the deviation from reference index was as follows. (1) We computed the participant average difficulty judgment across problems (range 1–10); (2) We subtracted 5 from (1); (3) We subtracted (2) to each individual participants' problem difficulty judgment (4) For each problem ($n = 30$), we computed the average of (3) restricted to the healthy participant group ($n = 25$). This is the reference difficulty score of each individual problem. (5) For each subject, we computed the sum of the square of the difference between (3) and (4). (6) The average across problems for a single subject is the subject's deviation from reference score.

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Table 3
Average and standard deviation for the experimental variables we considered. Variables are reported in each lesion group and in healthy control participants.

	MED	LL	RL	Patients overall	CTL
Deduction (accuracy)	0.78** (0.12)	0.79** (0.11)	0.85 (0.07)	0.80** (0.11)	0.88 (0.08)
Difficulty ratings	3.16 (1.19)	2.94 (1.33)	2.98 (0.87)	3.06 (1.14)	2.88 (0.88)
Deviation from reference	2.64* (1.63)	2.02 (0.95)	1.75 (0.65)	2.27 (1.32)	1.70 (1.20)
Difficulty-accuracy consistency	0.64* (0.33)	0.75 (0.26)	0.78 (0.21)	0.70 (0.29)	0.81 (0.29)
Digit span forward	5.39* (0.92)	5.70 (1.42)	6.00 (1.51)	5.61* (1.20)	6.07 (1.21)
Digit span backward	4.83 (1.20)	4.30* (1.06)	4.63 (0.92)	4.64 (1.10)	5.11 (1.45)
Token test	38.47 (2.06)	38.70 (1.34)	38.83 (0.98)	38.61 (1.64)	–

* $p<0.05$.
** $p<0.01$.

on the number of problems incorrectly answered. Because deductive accuracy was higher in controls than in some patient groups, it is possible that this introduced an inhomogeneity in the variance across groups. We checked whether a difference in variance existed among the patient subgroups. No such difference was found. For two reasons, this result suggests that medial prefrontal patients may have specific difficulties in following the overall structure of an elementary reasoning process. Left lateral patients, whose overall performance level was virtually the same as the medial group, showed no problem on the consistency measure. Also the other non-standard dependent variable (the "deviation from reference") we analyzed revealed the same pattern of a selective deficit in medial patients.

3.4. Deduction and verbal comprehension

We found no indications in the data suggesting that a comprehension deficit generated the pattern of impairments observed in our patients. All three frontal subgroups produced a very high score on the Token test, totaling on average more than 38 out of 40. Furthermore the score of the left lateral subgroup – in which it is most probable that a comprehension deficit could be found – was similar to that of the other patient subgroups. Finally, we tested the relationship between the Token test and accuracy on the deduction test by means of a regression analysis, partialling out the effect of the demographic factors. We found no significant correlation, whether in subgroup analyses or in the patients overall.

3.5. Deduction and verbal working memory

Frontal patients performed significantly worse than control subjects on digit span forward ($F(1,59)=3.52$, $p=0.033$), but not on digit span backward ($F(1,59)=2.47$, $p>0.05$). In the subgroup analysis, an impairment on digit span forward emerged only in medial patients ($F(1,41)=5.29$, $p=0.013$), while left lateral and right lateral patients were spared (respectively, $F(1,33)=1.36$, $p>0.1$, and $F(1,31)=0.23$, $p>0.1$). By contrast, only left lateral patients were impaired in the backward digit span test (left lateral: $F(1,33)=3.13$, $p=0.043$; right lateral: $F(1,31)=0.99$, $p>0.1$; medial: $F(1,41)=0.46$, $p>0.1$).

In order to explore whether the pattern of impairment found in the left lateral and medial subgroups could be due to deficits of verbal working memory, we re-analyzed the deduction test by considering only those patients with spared working memory (WM+). To be confident that the memory span of the selected patients was preserved, we used a conservative criterion. We excluded patients who scored lower than the median of our controls in either digit span backward or digit span forward. We first considered digit span backward. In it, the selected patients had a median equal or higher than 5. Even on the light of this conservative criterion, medial WM+ frontal patients produced significantly more errors in the deduction test ($F(1,33)=4.786$, $p=0.018$). By contrast, left lateral WM+ patients were not impaired. Furthermore, for all the three scores related to difficulty judgments, the pattern of results for the medial and left lateral WM+ subgroups mimicked the full subgroup analyses (Table 4). Thus, in the medial WM+ subgroup both the deviation from reference and the consistency with accuracy indexes were respectively higher and lower than the control group (deviation index: $F(1,33)=3.85$, $p=0.030$; consistency: $F(1,29)=3.03$, $p=0.046$). The average difficulty judgment was not different from controls ($F(1,33)=0.31$, $p>0.1$). Instead, none of the scores of the left lateral WM+ subgroup was different from controls.

The same findings were obtained by using digit span forward. We applied the same conservative criterion used for backward span, thus excluding patients with a forward span score lower than the median of the control subjects (median = 6). Medial patients with spared verbal short-term memory ($n=10$) were still impaired in the deduction test ($F(1,31)=7.11$, $p=0.006$), unlike left lateral patients ($n=4$; $F(1,25)=0.20$, $p>0.1$). Medial patients with spared short-term memory also had a deviation from reference index higher than control subjects ($F(1,31)=3.31$, $p=0.039$).

3.6. Nonlogical factors in problem assessment

As in every reasoning experiment which employed problems with content, extra-logical factors may have influenced participant's performance and perceived difficulties in our experiments. While no absolute control of all such factors is possible, we can exclude that major nonlogical aspects of them account for the results we report.

One major factor is problem length, which has been reported to correlate highly with both latency and subjective difficulty (but not with accuracy) in the type of problems we investigated (Braine et al., 1984). We checked how problem length correlated with the measures we report. Correlations between problem length and latencies ranged from .21 to .26 in controls, in patients overall and in patients' subgroups. Correlations between problem length and difficulty judgments ranged from .12 to .18. Although the limited number of errors makes correlations with accuracy scores meaningless, it can be observed that the average length of problems with correct responses ranged from 46 to 47 words, while the average length of problems with incorrect responses was 46 words. Clearly, problem length had no effect on our measures, whether in patients or in controls.

Although controlling for content-related factors is more difficult, a way to estimate major content effect is to compare how control participants reasoned with our stories and how participants reasoned with logically identical stories filled with different content, as well as with problems presenting their simple logical skeleton deprived of any content. Data by Bonatti and Viti (in preparation) allow us to make both such comparisons. In two experiments Bonatti and Viti employed, among others, the problem structures we used, but with stories having different contents. Problems were presented either in written or oral form, allowing the influence of the mode of problem presentation we chose to be assessed. Although the near-ceiling performance in the Bonatti and Viti study makes it impossible to compare accuracy scores

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Table 4
Average and standard deviation for the experimental variables we considered, sorted by performance on digit span backward. Each lesion group has been split depending on the performance on digit span backward. WM+ refers to lesion subgroups with a spared digit span, while WM− refers to lesion subgroups with an impaired digit span.

	MED		LL		RL		Patients overall	
	WM+	WM−	WM+	WM−	WM+	WM−	WM+	WM−
N	12	6	5	5	5	3	22	14
Deduction (accuracy)	0.82 (0.10)	0.72 (0.13)	0.88 (0.06)	0.70 (0.06)	0.85 (0.06)	0.83 (0.09)	0.84 (0.09)	0.74 (0.11)
Difficulty ratings	3.06 (1.14)	3.38 (1.37)	2.65 (1.08)	3.22 (1.61)	2.92 (0.98)	3.07 (0.86)	2.93 (1.06)	3.25 (1.28)
Deviation from reference	2.72 (1.86)	2.48 (1.16)	2.21 (1.34)	1.83 (0.37)	1.54 (0.56)	2.11 (0.74)	2.33 (1.57)	2.17 (0.86)
Difficulty-accuracy consistency	0.63 (0.36)	0.66 (0.31)	0.83 (0.24)	0.67 (0.27)	0.75 (0.25)	0.82 (0.17)	0.70 (0.31)	0.70 (0.26)

directly, difficulty judgments can be compared. For the problem structures common to both studies, we found a .8 correlation between the judgments of our controls and those of Bonatti and Viti's participants. The correlation was as high with problems both presented orally and in written forms in the complementary study. Even more strikingly, we also found a .65 correlation between difficulty scores for our problems and the difficulty judgments given by Bonatti and Viti's participants solving logically identically structured problems lacking any content.

Overall, these analyses suggest that major extra-logical factors cannot account for the results we report. Therefore, we feel confident to conclude that the deficits we report are due to specific logical impairments of the subgroup of patients we tested.

4. Discussion

Elementary deduction is the ability to draw conclusions without reflection from explicit or implicit premises, on the basis of their logical forms. This ability is involved in many aspects of human cognition, such as belief fixation, conversational exchanges, and internal thinking processes. Clarifying its nature and its neural basis is an important task, which researchers only began to approach recently.

The present study had two main objectives. The first was to provide neuropsychological evidence about the involvement of different regions in the frontal cortex in an elementary deductive task. The second aim was to provide evidence for our hypothesis that elementary deduction, although psychologically primitive, is likely to be organized into functional subcomponents possibly involving different cerebral areas.

4.1. *Elementary deduction and frontal lobes*

Our results show that the frontal cortex is heavily involved in elementary deduction. However, not all frontal sub-regions are involved equally. Only lesions to the left lateral and the medial frontal cortices impaired patients' abilities at solving elementary deduction problems. By contrast, patients with lesions to the right lateral frontal cortex did not produce more logical errors than controls. This pattern of results suggests that, just as the right lateral cortex is not necessary for the carrying out of inductive reasoning tasks (Reverberi, D'Agostini, Skrap, & Shallice, 2005; Reverberi, Lavaroni, Gigli, Skrap, & Shallice, 2005), it is also not necessary for elementary deduction.

Our findings cannot be explained by a mere verbal comprehension deficit affecting the left lateral and medial, but not the right lateral, patients. Two main aspects of the results make this interpretation unlikely. First, all the patient subgroups performed almost at ceiling in the Token test. Second, in the deduction test we found no correlation between Token test and Accuracy scores. Had a verbal comprehension deficit caused the differential performance of our patients' subgroups, such a correlation would be expected to occur.

The specific involvement of left lateral and medial frontal cortex in elementary deductive reasoning is broadly consistent with existing neuroimaging evidence. Activation foci in the left lateral frontal cortex while solving deductive problems have been reported. In particular, left lateral activations were observed in all neuroimaging studies using propositional deductive material (Monti et al., 2007; Noveck, Goel, & Smith, 2004; Reverberi et al., 2007; Reverberi et al., submitted for publication) and quantified deductive material (Goel et al., 2000; Goel & Dolan, 2003, 2004; Osherson et al., 1998; Reverberi et al., submitted for publication). However, activation of medial frontal regions has been found only in two studies on propositional deductive reasoning (Monti et al., 2007; Noveck et al., 2004), and in no study using quantified deductive problems. Although methodological differences and the range of analytical tools employed in the various studies could explain such differences (see Monti et al., 2007), it is also possible that different classes of deductive tasks elicit different neural processes. Thus, the design of our study (like that of Monti et al., 2007) might be more sensitive to aspects of deductive processing that specifically tax the medial cortex. We favor this interpretation, which we discuss further in the next section.

Besides revealing the critical roles of the left lateral and the medial frontal cortex in elementary deduction, our results also help to differentiate the specific functional roles of the two cortical regions. In both the left lateral and the medial patients, the accuracy score on the deduction task was low. However, the two groups clearly behaved differently in other related measures of deductive ability. For left lateral patients, the accuracy score in the deduction task parallels performance in working memory tests (as measured both by digit span forward and, more critically, digit span backward). While left lateral patients with working memory deficits had severely flawed deductive abilities, those with spared working memory were as good as healthy controls. Instead, in medial patients, a spared working memory did not suffice to preserve deductive abilities (Table 4). Furthermore, medial frontal patients did not judge the difficulty of the deductive problems correctly (as both the higher deviation from reference scores and the lower difficulty-accuracy consistency indices show), whereas left lateral patients were able to understand and judge the difficulty of a problem just like healthy controls. This pattern also holds for those left lateral patients who had both a reduced working memory span and were severely impaired in the deduction task (Table 4).

The differential pattern of performance of the left lateral and medial groups suggests that a multi-factorial model of deductive competence, such as the one we described in the Introduction, is best suited to account for the results. Following that model, we hypothesize that the overall performance of medial patients could be explained in terms of a deficit in identifying and representing the overall structure of the proof required to solve a deductive problem. This deficit would produce both the inability to judge the difficulty of a problem and the observed reduction in accuracy. Over and above this deficit, some medial patients can also have working memory problems, which can lead to a further deterioration in their performances. This interpretation of the medial impairment

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is consistent with theoretical proposals linking anterior prefrontal cortex (lesioned in almost all our medial frontal patients) to the ability to hold goals in mind while processing intermediate subgoals (Koechlin & Hyafil, 2007; Koechlin et al., 1999; Burgess et al., 2000).[3] The proposed functional role of the anterior medial frontal cortex also helps explain why it was activated in only two neuroimaging studies of propositional reasoning. Building and updating a representation of the overall structure of a proof would be most critical only when the deduction needed to derive a conclusion cannot be obtained by applying a single primitive rule, but requires a structure of a certain complexity. In the two studies reporting medial cortex activation (Monti et al., 2007; Noveck et al., 2004), the stimuli employed required inferences such as *modus tollens*, that is, inferences that go beyond natural elementary reasoning and demand the reasoner to build a *reductio ad absurdum* proof. Likewise, neuroimaging studies using simple, one-step propositional problems (such as *modus ponens* or disjunction elimination), or simple syllogisms, did not obtain medial activations. Further evidence is needed to test this interpretation.

Unlike medial patients, the overall performance of left lateral patients cannot be due to a meta-deduction deficit, as the meta-deduction judgments of left lateral patients were fully normal. Instead, the fact that only those left lateral patients with a working memory deficit were impaired in carrying out deductions (Table 4) suggests that their working memory deficit is at the origin of their deduction deficit. Specifically, according to this hypothesis, the left lateral cortex contributes to deductive reasoning by providing the memory space necessary to build the representations created while deriving deductive conclusions. Absence or disruption of such working space could impair deductive abilities even if the basic repertoire of deductive and meta-deductive procedures necessary to derive a conclusion is preserved.

In our study we did not find localizing evidence for one of the cognitive components of elementary deduction we postulated, namely that of applying single elementary steps in the generation of valid deductive proofs. In the Mental Logic framework, this component could be described as the application of the appropriate elementary rule of inference during proof construction. In the Mental Model framework, it would correspond to the processes involved in integrating premises during first model construction. In order to demonstrate a specific deficit for such a cognitive component, we should observe a lesion subgroup with both working memory and meta-deductive monitoring abilities spared, but who is impaired in carrying out deductive problems. We did not observe such a pattern. We can advance three explanations for the failure to find it.

The first explanation follows from the way that that working memory and meta-deductive monitoring measures doubly dissociate (see Table 4, deviation from reference in the medial group WM+ and left lateral group WM−). Thus, we provided strong evidence for the separation of these two components. However, it is not impossible that a single mechanism generates deductive reasoning steps and keeps track of its complexity. That is, possibly meta-deductive monitoring and proof construction originate from the same mechanism. A second possible explanation would be that although the generation of valid conclusions is an independent cognitive function, its neural basis is very similar to that of working memory or meta-deductive monitoring; perhaps even the same structures may support different types of operation (Duncan & Owen, 2000). A third explanation would be that the process of proof construction is separable both functionally and anatomically, but originates from brain areas that we have not studied in this article. Our data are compatible with all three explanations.

However, ongoing and completed neuroimaging studies can help one to choose between them. Reverberi and collaborators (Reverberi et al., 2007, submitted for publication) explored the neural basis of proof construction in elementary deductive problems. In those studies, participants solved only single-step deductive problems, thus reducing or eliminating the need of building and updating representations of the structure of a proof. Furthermore, particular care was devoted to equate the working memory requirements of the experimental and the baseline conditions. In particular, Reverberi et al. (submitted for publication) studied the critical time window of interest, after participants were presented with identical sets of premises. They were required either to generate a new deductive conclusion (experimental condition) or to encode and remember a novel sentence (baseline condition). Across the baseline and experimental conditions, participants had to remember sentences of equal length and syntactic complexity, and to update the working memory content in the same way. Thus, in both conditions the contribution of working memory was equated and hence could not account for the results. During the generation of deductive conclusions, activation of the left lateral frontal cortex – mainly of Brodmann Area 44/45 – was observed in both studies. Interestingly, although these activations were in the left lateral frontal cortex, they do not overlap with the lesion maps of our left lateral patients, because we excluded from our sample patients with moderate to severe aphasia, a probable outcome of BA44/45 lesions (the Broca's region).

These elements favor the third explanation we advanced. We thus suggest that the left lateral frontal cortex implements two functions relevant for deductive reasoning. One region, a more posterior one, handles step-by-step proof construction, whereas a more anterior region contributes to the temporary storage of the relevant representations. In the current study, we could only observe the effects of lesions to this more anterior left region. This hypothesis provides a series of empirical predictions that we are currently studying.

5. Conclusions

The present study explored the contribution of specific regions of the human frontal cortex to elementary deduction. We explored the view that even elementary reasoning involves multiple functional components, with possibly different localizations.

We found that left lateral and medial frontal cortices have a critical role in elementary deduction, whereas, lesions to right lateral cortex did not affect deductive competence. Furthermore, by also evaluating working memory span and meta-deductive monitoring, we were able to differentiate the functional roles of the left lateral and medial cortex.

The overall performance of our patients supports a model that separates elementary deductive competence into three more basic cognitive components: one that generates deductive conclusions (application of deductive rules for proof construction), one that keeps track of the overall structure of the proof being constructed, and one that holds the necessary intermediate representations during proof construction. We provided evidence for the role of two of these components: the meta-deductive representation of the structure of the proof, which we showed to be linked to the medial frontal cortex, and the working memory component. Thus, not only did we provide neuropsychological evidence for the multiple processes involved in elementary deduction, but we also showed that even the

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[3] In a few left lateral frontal patients ($n = 4$) the left anterior prefrontal cortex was also involved (BA10, accounting for only the 10% of the overall lesion size). Such minor involvement cannot explain the performance of the left lateral group. To control for this possibility, we run an ANOVA by excluding the four left lateral patients with a minor anterior prefrontal cortex involvement. Even so, the left lateral lesion subgroup still showed a selective impairment in accuracy judgments ($F(1,27) = 7.099$, $p < 0.01$).

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most basic deductive reasoning process is more complex than what the current theories of reasoning assume.

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Appendix A

Examples of problems presented to our participants. They span different difficulty ratings, from easy to difficult. The problems were originally presented in Italian.

A.1. Example 1 (rated 2.04)

This story is about Mary and Luke

P1 If Mary wants to talk to Luke, he answers that he is happy to do that.
P2 If she asks him to help her cleaning up the apartment, Luke also answers that he is happy to do that.
C If Mary wants to talk to Luke, or if she asks him to help her cleaning up the apartment, Luke answers that he is not happy to do that.

A.2. Example 2 (rated 4.00)

This story is about Rose and George

P1 It is not true that, as George claimed, Rose did not eat fish yesterday at the restaurant.
P2 George either didn't order fish, or left the fish on his plate.
C Yesterday, either Rose ate fish and George did not order it, or Rose ate fish and George left his fish on the plate.

A.3. Example 3 (rated 5.63)

This story is about Jane, David and their friend Anne

P1 Either Jane will go shopping, or David will go playing golf.
P2 Jane will not go shopping.
P3 If David goes playing golf, or if he decides to clean his car, Anne will not go visit Jane and David at teatime.
C Anne won't go visit Jane and David at teatime.

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