CHAPTER ONE

The Probable and the Possible at 12 Months: Intuitive Reasoning about the Uncertain Future

Nicolò Cesana-Arlotti*, Erno Téglás, and Luca L. Bonatti*,[1]**

*ICREA, Universitat Pompeu Fabra, Barcelona, España

**Cognitive Development Centre, Central European University, H-1015 Budapest, Hungary

[1]Address correspondence to: Luca L. Bonatti, ICREA, Universitat Pompeu Fabra, C. Roc Boronat, 138, Edifici Tanger, 55.110, 08018 Barcelona, España. Email: lucabonatti@mac.com

Contents

Abstract

How do infants predict the next future event, when such a prediction requires estimating the event's probability? The literature suggests that adult humans often fail this task because their probability estimates are affected by heuristics and biases or because they can reason about the frequency of classes of events but not about the probability of single events. Recent evidence suggests instead that already at 12 months infants have an intuitive notion of probability that applies to single, never experienced events and that they may use it to predict what will happen next. We present a theory according to which infants' intuitive grasp of the probability of future events derives from their representation of logically consistent future possibilities. We compare it and other theories against the currently available data. Although the evidence does not speak uniquely in favor of one theory, the results presented and the theories currently being developed to account for them suggest that infants have surprisingly

Advances in Child Development and Behavior, Volume 43
ISSN 0065-2407,
http://dx.doi.org/10.1016/B978-0-12-397919-3.00001-0

sophisticated reasoning abilities. These conclusions are incompatible with most current theories of adult logical and probabilistic reasoning.

The facts in logical space are the world

L. Wittgenstein, Tractatus, 1.13

1. INTRODUCTION

Often we have no idea what will happen next. We do it more often than we think. One sign of proof is our survival, which requires us to anticipate future events. True, we are not the only living beings on earth, and so survival is not particularly indicative of any distinctive ability at predicting the future. However, humans do more than just survive: they radically modify their environment. Take a look at your surroundings and estimate how much of your environment is comprised of man-made things. Almost every single object in our natural environment—by now, cities, houses, offices, and not forests and prairies—and almost every single action we make drips with human inventiveness. This incredible richness is proof of the continuous, neurotic pressure to plan and invent new things, to think ahead at what happens if I do this and that, and to make and realize plans: an ability that we can trace back to the beginning of the human species (Amati & Shallice, 2007). We take all these for granted. Our question here is: if this ability is based, among other things, upon the ability to think about the future, how do we and when can we represent future states of affairs?

We should not expect to find a single answer to these questions. Certainly, as David Hume famously argued (Hume, 2000), often even when we think of simple physical events, experience is our guide to predict the future. I have seen the sun rising in the past, and on the basis of this repeated experience, I predict that the sun will rise tomorrow. Many organisms, besides humans, can learn from the regularities around them (Hauser, Newport, & Aslin, 2001; Toro & Trobalón, 2005). However, equally often we jump into the future with only a scant experience of the past. We can do it in different, nonexclusive ways. We may anticipate novel future outcomes because we possess hardwired systems that, in limited domains, generate future states of affairs. Or we may freely combine already acquired knowledge and information from different domains, although we have never experienced that particular combination. We may also anticipate the future in the absence of experience because we think that the next future event logically follows from what we know: if I know that if John meets Mary he will be happy, and I know that he will meet Mary tomorrow, I need no experience to anticipate that tomorrow he will be happy. And, given that (barring logical consequences) the future is the realm of uncertainty, we may predict future events because we have a sense of what is likely to follow. This paper mostly concerns these two notions—logic and probability—and their interrelations. We want to first present a theory about their relations. We will then present some relevant data about the origin of these abilities. Finally, we will sketch some future directions of our research.

2. CAN HUMANS REASON ABOUT THE PROBABLE AND THE POSSIBLE?

That humans can reason logically, or probabilistically, cannot be taken for granted. In fact, the bulk of the literature on adult human reasoning has been taken to support the opposite conclusion: the existence of logical and probabilistic abilities has been severely challenged. For logical reasoning, since Wason's famous work on the selection task (Wason, 1968), studies showing logical mistakes have flourished (e.g. Evans, 1989). At best, logical reasoning has been relegated to a secondary ability of minor importance. Thus, the widely held dual process account (e.g. Evans, 2003, 2008; Stanovich & West, 2000; Sloman, 1996) claims that two distinct cognitive systems underline reasoning: an evolutionary primitive (set of) system(s), providing preanalytic answers to problems (sometimes called System 1), and a more recent system by which humans can achieve abstract thinking (System 2). System 1, which we share with other animals, is considered to be at the origin of most problem solving. It is a fast, nonverbal, emotionally driven, associative, and intuitive source of responses to situations. It generates answers that are driven by biases, heuristics, or pragmatic factors and may thus lead humans to make many errors, when such nonlogical strategies are inappropriate. By contrast, System 2 is described as a uniquely human, verbal, explicit, serial, "rational" form of reasoning. Its advantage is to permit abstract reasoning and hypothetical thinking, but its disadvantages are many. It is slow, weak, easily overwhelmed by even minimally complex problems and extremely variable among individuals. This theory, as Evans wrote, "quite literally proposes the presence of two minds in one brain" (Evans, 2003, p. 5).

If such a theory is correct, there is no point in writing this paper. Not only are animals nonlinguistic beings but so are infants. Hence, by definition, under this theory, they would only possess one of the two minds: the intuitive, associative, and irrational mind. Fortunately, we believe that there are empirical data and principled arguments to show that this is not the case. For the moment, we only want to note that clearly the theory, which is based entirely on experiments with adults, has not considered its developmental implications. If System 2 is the explicit, logical, linguistic, and weak rational reasoning system, how would it ever enter within an infant mind which only contains System 1? The very same problems raised by Fodor against Piaget many years ago (Piaget, Chomsky, & Piattelli-Palmarini, 1980) would entirely apply to this theory, with equally devastating consequences.

To a first approximation, knowing how to deal with probabilities is even more fundamental than the ability to reason about logic in order to anticipate future events (but see later). But even in this domain, a long series of studies championed by Tversky and Kahneman seems to suggest that humans are very poor probabilistic reasoners (e.g. Tversky & Kahneman, 1974, 1981). When participants have to judge the likelihood of single future events, their responses are driven not by rational evaluations of what is likely to be the case but by heuristics and biases that may lead to serious mistakes. Even apparent alternatives to the "Heuristics and Biases" theory, such as the frequentist approach (Cosmides & Tooby, 1996; Gigerenzer & Hoffrage, 1995), share one point in common with it. They also consider human abilities at handling logic and probability as severely constrained. For Cosmides, humans can reason logically only in limited contexts of social exchange (Cosmides, 1989) but do not possess the ability to reason generally with logical rules because evolution could not have favored the selection of such a general-purpose reasoning mechanism.

These issues are inextricably linked with more general questions, spanning from the nature of the mind to the foundations of probability. Let us briefly see why. One of the main divides in the foundations of probabilities concerns the status of attributions of probabilities to single events. On the one hand, it looks very natural to say that I have a certain degree of belief that a future event will occur. That is, intuitively we seem to think that there is nothing wrong in saying that I am afraid that tomorrow it will rain. Translated in terms of beliefs, this statement corresponds to a belief, with certain strength, that tomorrow it will rain. In its turn, translated in terms of probabilities, this belief may be interpreted as a certain internal state in which we attribute a degree of probability to the future event "Tomorrow it will rain." Thus, beliefs could be thought of as subjective degrees of probabilities about single future events.[1] However, according to frequentists, the reality of the world is that tomorrow it will either rain or not rain: there is no sense in which tomorrow "it will 60% rain." Thus, so the argument goes, from a realistic standpoint, single-case probabilities are meaningless. If one wants to make sense of probabilities, the only way to do it is to treat probabilities as frequencies: the subjective, Bayesian understanding of probabilities is irredeemably subjective and, as such, should have no place in science. It should be clear now how far a question about single-case prediction leads us into very controversial areas, obliging us to take a standpoint on many vexed philosophical issues.

We can see the frequentist view as the psychological adaptation of the frequentist point of view in probability. According to this theory, because ontologically probabilities are frequencies, psychologically they can be understood only as collections of experienced events. Cosmides and Tooby are quite explicit about this. Our sense organs, so they argue, can only discern what can be observed and "the 'probability' of a single event is intrinsically unobservable." Thus, again, evolution could not have selected a mechanism computing single-case probabilities. What, instead, we can observe are "encountered frequencies of actual events" (Cosmides & Tooby, 1996, p. 15). Hence, frequency detection mechanisms, tracking collections of experienced events, could survive selective pressure but not systems predicting the probability of single future events. We do not want to comment on this evolutionary argument, which we believe to be seriously flawed. We only want to notice that, taking this perspective to its extreme consequences, we should conclude that our intuitions about the future are entirely dependent on our experience of the past.

This brief review offers a bleak picture of human reasoning abilities. The ability to draw logical inferences, to estimate the probability of the next future event, and to base our predictions on such estimates are landmarks of rational cognition. The fact that humans fail in both domains certainly does not allow us to be optimistic about the rationality of mankind. Yet, recent discoveries suggest that the bleak picture is only a partial reconstruction of the real richness of human cognition, and not because humans possess a weak System 2 that sometimes gets it right but because logical and probabilistic

[1] For the purpose of this discussion, it is not important to take a stand as to whether a Bayesian view of beliefs is a good account of beliefs. Our point here is only that there are ways to naturally read degrees of beliefs as probabilities assigned to propositions, quite independently of the computations of frequencies with which a certain proposition turns out to be true.

abilities are much deeper inside the fiber of the human mind. First, a wide set of modeling and experimental studies shows that adults spontaneously make predictions that are well captured by Bayesian models (e.g. Griffiths, Kemp, & Tenenbaum, 2008; Griffiths & Tenenbaum, 2006; Tenenbaum, Griffiths, & Kemp, 2006). Now, in a Bayesian approach, being rational is being able to predict the next future event from a priori hypotheses about what will happen and from the ability to revise them based on what really happened. Such predictions cannot be formulated without the adequate logical and probabilistic inferential mechanisms—the very same ones that the heuristics and biases and the dual process theories assume humans do not possess.

Second, recent studies on biased reasoning show that participants process correct normative information (logical or probabilistic) even in tasks in which they are influenced by heuristics or in which they are under cognitive load. For example, De Neys and Schaeken (2007) showed that the amount of pragmatic interpretations of the meaning of utterances reduces in favor of their logical interpretations under cognitive load. Likewise, in conditions in which beliefs conflicted with reasoning structure, memory access to words that were associated with beliefs was impaired, suggesting that at a fairly low level reasoning inhibits the retrieval of nonlogical beliefs (De Neys & Franssens, 2009). These data suggest that logical abilities do not reside only in the more effortful, abstract, and frail component of the cognitive system but are as intuitive and immediate as System 1 according to the dual process theory.

Third, previous and recent data, which the works mentioned above nicely complement, show that adults implicitly draw elementary logical inferences when thinking about everyday situations (Braine, O'Brien, Noveck, Samuels, Lea, Fisch et al., 1995; Lea & Mulligan, 2002; Lea, O'Brien, Fisch, & Noveck, 1990), even when they are unaware of doing so (Reverberi, Pischedda, Burigo, & Cherubini, 2012). Indeed, recent neuropsychological data show that specific patterns of neural activities can predict participants' sensitivity for the elementary logical structure of stories and formal problems, but no neural pattern predicts the tendency to rely on (at least some) heuristics (Reverberi, Bonatti, et al., 2012; Reverberi, Shallice, D'Agostini, Skrap, & Bonatti, 2009). This result suggests that, if anything, logical processes in adults occupy a more central role than heuristics.

The point we want to make is that from our current understanding of reasoning a picture emerges that is much richer than the one advertised by most literature on adult reasoning. It suggests that theories that consider logical and probabilistic reasoning processes of secondary importance in our mental life, such as the dual process theory, cut the pie in the wrong way (De Neys, 2012).

The dual process theory describes human reasoning as characterized by the opposition between intuitive, fast, and immediate heuristics and a slow, verbal, and frail logical reasoning. There is no such opposition. Instead, already at the level of intuition—as it were, deep down in the machine—humans seem to possess basic logical and probabilistic inferential devices. If so, then it is quite possible that the same machinery is already available at early, prelinguistic stages of knowledge representation, as a foundation of the way humans think, organize their plans, and use their knowledge to predict future states of the world. We now want to inspect this possibility and offer a theory about what this machinery could provide to the overall efficiency of our cognitive processes.

3. INFANTS' REASONING ABILITIES: DOMAIN-SPECIFIC MECHANISMS, GENERAL SYSTEMS OF INFERENCES, AND FUTURE PREDICTIONS

The last three decades of research have revealed the existence of several domain-specific reasoning mechanisms that may guide infants in predicting future situations in limited domains. Notably, we know that infants understand basic physical principles (Baillargeon, Spelke, & Wasserman, 1985; Spelke, Breinlinger, Macomber, & Jacobson, 1992). They interpret agents' behaviors as goal-oriented (Woodward, 1998, 1999) optimal solutions toward the realization of their intentions (Gergely & Csibra, 2003; Gergely, Nádasdy, Csibra, & Bíró, 1995). They possess dissociable systems for precisely representing small arrays of individual objects and imprecisely representing large quantities (e.g. Feigenson, 2005; Feigenson & Carey, 2005; Feigenson, Dehaene, & Spelke, 2004; Xu & Spelke, 2000).[2]

All the abilities we mentioned above have sometimes been described as independent modules, or core domains, not necessarily interconnected.

[2] While such abilities might be used proactively to predict the continuation of an event, we have surprisingly little evidence that infants do indeed use such domain-specific knowledge to proactively predict future events. The violation of expectation method, upon which most of our knowledge about infants' domain-specific abilities rests, allows us to conclude that infant are surprised at a given (unexpected) outcome but not that they predicted the opposite, expected, outcome. At least in the case of agency, we do know that infants do not need to experience the end state of an action in order to attribute a goal to the actor and infer a future goal state (Southgate & Csibra, 2009), but this evidence is much less widespread than what one would need to prove that domain-specific systems are actively predicting future states of affairs. However, for the purpose of this discussion, we will not distinguish surprise at an unexpected outcome and prediction of the expected outcome, although they are possibly distinct.

Indeed, according to some authors, it is the emergence of language that allows humans to glue together originally separated, possibly noncommunicating, information processors into a unitary cognitive architecture (e.g. Carruthers, 2002; Mithen, 1996; Spelke, 2003). One consequence of this thesis is that prelinguistic infants may not naturally integrate information coming from different domain-specific mechanisms to reason about the future optimally.

Yet infant cognition is not a simple collection of independent modules. Infants also possess domain-general cognitive abilities. Of particular importance is the ability to extract absolute and relative frequencies of different kinds of events, such as speech events, or visual stimuli (Fiser & Aslin, 2002; Saffran, Aslin, & Newport, 1996). This ability may help explain how infants solve several learning problems in various domains. However, frequency computations can guide future predictions only through past experiences and hence are mute to the problem of whether infants can reason about the future in the absence of past experience. Abilities that are more difficult to explain on the basis of simple past experience have been discovered by Xu and Garcia (2008) and Téglás, Girotto, Gonzalez, and Bonatti (2007). Although these studies have important differences that we will discuss below, both show that infants intuitively make inferences about probabilities that do not require frequency detection mechanisms and cannot be simply explained on the basis of previous experience.

Xu and Garcia (2008) showed that infants seem to have an intuitive grasp of the relation between samples and populations. Given a sample, infants can infer the distribution of the population from which it had been drawn. Conversely, given a full population, they expect a sample drawn by it to reflect the distribution of the population. They are willing to infer the statistical relations between samples and population only when samples are randomly drawn (Xu & Denison, 2009), reinforcing the view that infants have a basic grasp of random processes—precisely what, according to traditional studies in the heuristics and biases tradition (Gilovich, Vallone, & Tversky, 1985; Kahneman & Tversky, 1972; Tversky & Kahneman, 1993), adults often fail to display. While certainly the kinds of situations tested by Tversky and Kahneman are more complex, the contrast is intriguing. Xu and her colleagues also showed that infants are able to integrate information from different sources in their inferences. At 11 months, they can use information about other people's intention in order to infer that a sampling process is random (Xu & Denison, 2009), and at 20 months, they also consider a violation in the randomness of a drawing as evidence for the preferences of agents (Kushnir, Xu, & Wellman, 2010). These findings point at the fact that infants' reasoning is more structured than what a passive, purely data-driven mechanism would predict. General-purpose mechanisms transcending individual core domains seem to be available early in development, enabling infants to rationally exploit the relevant source of information when reasoning about future states of affair.

While these results witness how rational infants can be in drawing inferences in conditions of uncertainty, they do not clarify another fundamental issue to understand whether infants can predict future events: can they estimate the likelihood of single future events in the absence of experience? This is what, according to frequentist theorists, humans cannot do. Téglás et al. (2007) explored this issue and argued that 12-month olds can do exactly that. Téglás and colleagues showed infants a simple situation in which three yellow objects and one blue object bounced inside a container with an opening on its lower side, as in a lottery machine (Fig. 1.2, left). Objects could be grouped into classes identified by the shape and color of the objects. After a period in which the objects bounced inside the container, an occluder covered its contents, so that the movements of the objects could not be seen. Then, before the end of the occlusion period, one of the objects exited the container. Finally, the occluder faded out and infants could look at the final scene. In the absence of any other information, a sense of probability would lead one to expect that one object of the most represented class would exit the container. Indeed, infants looked longer at the improbable outcome, in which the single object exited the container, signaling their surprise at an improbable single outcome.

Crucially, in this experiment, infants could predict the next single future event without having ever experienced it. In order to explain how infants could do that, it is not sufficient to postulate that infants have simple frequency detection mechanisms that track distributions of traits in samples and populations. Even in a frequentist view, infants could grasp such relations, and yet be unable to make single-case predictions about inexperienced outcomes. Something more is needed: an intuition of the probability of the next future event, that is, exactly the kind of intuition that, according to both the frequentist and the heuristics and biases views of human reasoning, adults do not possess.

4. A THEORY OF PROBABILISTIC REASONING: FROM LOGICAL REPRESENTATIONS TO SINGLE-CASE PROBABILITIES

We have argued that evidence exists that infants have an intuitive understanding of probabilities that is much more developed than most current theories of adult reasoning would incline one to think. However, we have made no mention of infants' logical abilities. The reason is simple: there is no information on this topic. What we want to do now is to speculate on how a theory of logical abilities could also explain infants' probabilistic reasoning. We first explicate the theory, and then we will discuss some of its consequences.

Let us ask the following question: How would a mechanism estimating the likelihood of single future events in the absence of experience look like? Here is a possible answer. Consider, as an example, the stimuli used by Téglás et al. (2007): a lottery-like container with three yellow objects and one blue one randomly bouncing inside it (Fig. 1.1A). One can consider it simply as it is—a container with four moving objects. However, the scene can also be represented in a modal way. Beyond its "face" appearance, the scene also individuates a series of logically possible future states of affairs: one in which a blue object exits the container and three in which a yellow object exits (Fig. 1.1B). We need no experience to conceive of such possibilities, provided that we can represent the logical space defined by the scene, compatible with some basic properties of our physical world such as object permanence and solidity. This modal nature of scenes and objects, we believe, was behind Wittgenstein's intuition that "the world is defined by the facts in logical space": metaphysically, a fact is already located in a logical space. Psychologically, we suggest, a fact is conceived as already carving possible future worlds that are compatible with it.

Thus, suppose that when infants look at a scene, not only do they represent the events and the classes of objects it contains (Fig. 1.1A) but they also construct its possible future outcomes (Fig. 1.1B). Suppose further that such possible outcomes can be coded by an appropriate numerical representation (Fig. 1.1C)—whether it be arrays of individual possible occurrences or a representation of the ratio between classes of possible events, regardless of their precise number. Then, under the assumption that all such logically possible outcomes are equiprobable, infants could also represent and estimate the probability that a single outcome (such as "a blue object exits the container") will occur (Fig. 1.1E) by comparing the number of

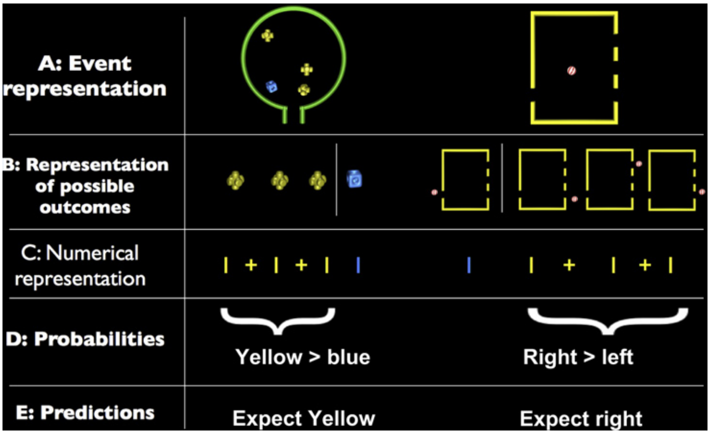

Figure 1.1 From possibilities to probabilities. A representation of how infants might reason about the probability of a single future event. When inspecting a scene (A), infants create a modal representation of it, hence representing its possible future outcomes (B). If this representation can access infants' numerical systems (C), the probability of the next future outcome can be computed independently of any experience (D). Such a computation can be the basis for predicting the next more likely single outcome (E). The scenes are schematic reproductions of the kinds of events presented to 12-month olds and older children in the studies described in the article (Téglás et al., 2007). In the first scene, the number of objects of different classes affords the cue to compute possible next states of affairs. In the second scene, the frame containing the single object offers the cue to project possible future outcomes. For color version of this figure, the reader is referred to the online version of this book.

possible outcomes in which the events of each kind take place (Fig. 1.1D). In short, we propose that a logical sense of possibility is the foundation of an intuition of the probability of single future events.

That the psychological concept of probability may derive from the concept of possibility was proposed by Johnson-Laird, Legrenzi, Girotto, Legrenzi, and Caverni (1999) within the framework of the mental model theory (Johnson-Laird, 1983). Technically, their theory of modal reasoning was an extension of the mental models' theory of propositional reasoning (Johnson-Laird, Byrne, & Schaeken, 1992). The soundness of this theory can be doubted (Bonatti, 1994; O'Brien, Braine, & Yang, 1994) and with it the soundness of its modal version. However, the intuition that the concept of probability depends on the concept of possibility is independent of the particular implementation that the theory of mental models gave to it and, in our opinion, remains entirely valid.

The view that the possibilities afforded by the logical space circumscribed by the scene, and not past familiarity with its outcomes, is at the basis of our intuitions of probability predicts that infants will have expectations about future events independently of frequency computations or of the relation between populations and samples. Furthermore, the view predicts that, all other things being equal, an intuition of probability will be unaffected by biases. All these, of course, must be proved.

5. INFANTS' EXPECTATIONS ABOUT THE PROBABLE FUTURE

There is a difficulty we must examine, before seeing how the theory might work. Is it possible to even get the idea of studying infants' intuitions about possibilities and probabilities off the ground? One immediate objection that would make our proposal not viable can be quickly formulated as follows: how could infants even figure out the space of logical possibilities? How are they supposed to identify what may be going to happen next? Will all objects exit the container? Only one? Perhaps two? Or will they all disappear or fly away? In principle, every scene is compatible with an infinite set of possibilities, out of which only few are relevant for the appropriate predictions. If we can speak and understand language, then delimiting the space is very simple: we can just tell what the relevant space is. We can ask, "What is the probability that the next object that comes out will be blue?," thus selecting some among the many possible relevant outcomes within a scene that must be considered in order to answer the question. Language is a very powerful tool that acts as selector for the relevant problem space. We have no such luxury with infants. They observe the world as it is, not as we describe it, and world scenes per se contain no explicit cue to the information relevant to constrain the space over which to reason about future events.

While ultimately general solutions to such questions are as hard as the frame problem, if infants can be cued to attend to a relevant solution space, then the problem is not unsolvable. Indeed, research in our laboratory suggests that an appropriate familiarization can be used (and must be used) to focus infants' attention to the relevant outcomes (Téglás & Bonatti, 2008). Thus, if infants are familiarized with one particular outcome among the many possible outcomes of a scene (in our example, if they are familiarized with a lottery-like scene in which one and only one object exits the device), then they will reason about the probability of the expected outcome. If, instead, infants are familiarized with the same situation, but no final outcome is ever shown, then they will not focus on the outcome of interest during the test phase and they will not form probabilistic expectations related to it. Infants must be cued to the relevant outcomes of a scene in order to reason about its possible future continuations. Also this behavior is entirely rational: if there is no way to represent the logical space of possible outcomes, then there is no way to predict which outcome will be more likely. Of course, the familiarization must give information about what kind of outcome to expect in the test scenes but not about its probabilities. In the case of the studies by Téglás and his colleagues, for example, infants were familiarized with four lottery-like scenes that contained objects of two classes of equal cardinality, so that the exit of an object of either one or the other class was equiprobable. In each of these familiarization movies, they saw one object (always a different one) exiting the container. Thus, although infants were familiarized with the outcomes of the scenes, they did not possess information leading them to expect one particular outcome. It could be said that also these familiarizations count as "previous experience" and, therefore, that frequentists are right that reasoning about the future requires past experience. We do not feel that this counter-objection describes the situation correctly. All the previous experience given in the experiments we described contains no information to bias infants to expect one particular outcome. Thus, these observations cannot be the basis of infants' responses when they are presented with the test situations, in which the class distribution of objects inside the containers is unbalanced. If anything, this brief familiarization might be used by infants to form expectations about the equiprobability of outcomes. Hence any outcome in the test phase (whether being the exit of an object of the most numerous class, or an object of the less numerous class) should appear as equally surprising. This is not, however, how infants react.

Here, we will not pursue the interesting question about the relation between experience, reasoning, familiarization, and test in experiments any further. In this context, we only want to make the point that infants can be cued into the dimension of what constitutes a relevant outcome, even without language. Language, we submit, is a useful tool to trim the infinite amount of possible continuations of a scene to the relevant class of outcomes. In this property resides its power to help us reason. But it is by no means necessary to reason logically or probabilistically. Provided that infants are cued into the dimension that makes a particular outcome the relevant one, then we can also probe how and whether they represent the relevant future possibilities.

In the experiments by Téglás et al., infants were first primed to the relevant solution space by movies that showed one single object exiting the container—a condition for the experiments to succeed. Then they saw movies terminating with the "probable" and "improbable" outcomes we described (Fig. 1.2, Scene A). As recalled, infants looked significantly longer at the improbable outcome, when the single object exited the container after an occlusion period, than at the more probable outcome in which one of the three identical objects exited. Importantly, Téglás et al. also showed that when a bar in the middle of the frame made it physically impossible for the three identical objects to reach the exit, infants inverted their preferences, looking longer at events in which one of them would exit (Fig. 1.2, Scene B). According to the theory of intuitive probability we are proposing, without the need to previously experienced distributions, infants naturally expect the more probable outcome based on the possible logical outcomes

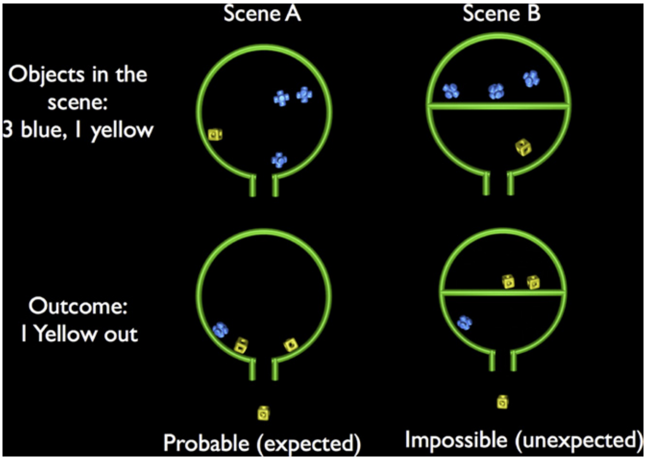

Figure 1.2 From probabilities to impossibilities. Both scenes have the same outcome: a yellow cubic object exits the container. However, in Scene A, the outcome is the most probable one, whereas in Scene B, it is impossible. Infants look at the outcome of Scene B longer when they find a cubic yellow object than when they find the single blue object, but look at the outcome of Scene A longer when they find the single blue object than when they find a cubic yellow object (Téglás et al., 2007). However, the configurational and low-level perceptual properties of the outcomes of Scenes A and B are highly similar. This inversion in looking pattern suggests that infants can consider the probability of an outcome just as efficiently as they consider its physical possibility, and are able to use the most efficient cues afforded by a scene to ground their expectations. For interpretation of the references to color in this figure legend, the reader is referred to the online version of this book.

of a scene. In the case of the simple lottery experiment, the (relevant) possible outcomes are the single outcome case in which a blue object exits the container and the three outcomes in which a yellow object exits. When a bar blocks the exits of the three yellow objects, then the only possible outcome is that in which a blue object exits the container. The computation we sketched above predicts exactly the pattern of inversion in looking time found by Téglás et al. (2007).

The fact that infants inverted their looking behavior when they saw the container with or without a bar in the middle also allows us to drive home another point concerning the relation between reasoning and biases at the origin of cognition. We know little of what a bias could be in an infant mind. However, if low-level factors such as perceptual grouping, or possible least effort strategies such as tracking the minimal number of objects, or intrinsic preferences such as a penchant for single-object outcomes count as proto-heuristics, then the inversion in looking patterns excludes that infants' reactions could be due to them. Of course the fact that infants' responses are not led by such proto-heuristics does not exclude that other, more complex problems require solutions based on heuristics. For example, the problems tested by Tversky and Kahneman, where the effect of heuristics appears, are far more complex than those studied by Téglás et al. (2007). Because it is almost impossible to test the same problems with 12-month-old infants, the issue is difficult to explore. However, the results of Téglás et al. at least suffice to establish that simple heuristics that could play a role in infants' reactions do not necessarily lead infants' reasoning astray. Thus, they open the possibility that the heuristics and biases present in adult reasoning are not immutable features of the human mind as the dual process theory holds. Instead, they may be spurious by-products of complex interactions between experience and mental mechanisms during development, rather than the product of our evolutionary history. Their explanation can be approached developmentally by studying their origins and the conditions under which they are formed.

6. INTUITIVE STATISTICS AND LOGICAL INTUITIONS OF PROBABILITIES: CONFLICTING OR COMPLEMENTARY EXPLANATIONS?

We proposed to explain the result of Téglás et al. by assuming that infants represent the (relevant) logically possible outcomes of a scene and, from them, form expectations about the most probable next future event. However, other alternative explanations are possible. In particular, we see an account along the lines of the theory proposed by Xu and Garcia (2008) that only appeals to intuitive statistics. Such an account would not need to postulate the representation of a space of future possibilities, and it may work equally well.

According to the intuitive probability proposal, infants represent a problem space as a small set of possible, yet never occurred, mutually exclusive events. Instead, an explanation based on intuitive statistics would consider the estimation of the probability of an event not yet encountered in terms of its relationship with an actual, perceived, distribution. Because in lottery experiments by Téglás et al. infants always have the full population in front of them, it could be possible to explain the results in terms of intuitive statistics as follows. Suppose that infants consider the objects inside the container (say, three yellow and one blue) as the full population of reference and the exit of one object from the container as a randomly drawn sample extracted from that population. Then, the exit of one yellow object will be less surprising because it would be considered as the draw of a one-object sample that better matches the distribution of the population than the draw of a one-object sample that contains the only existing blue object. Such reasoning does not require that infants conceive the space of the relevant possible outcomes. The full population is in front of them and they can directly compare the sample to the population distribution. This explanation might also account for the inversion of looking time between the situation in which every object could fall out of the container and the situation in which a bar blocked the yellow objects from reaching the exit. It might be possible to think that, when the bar isolates the three yellow objects, infants think that the population distribution from which the sample is drawn changes and consists only of the single blue object. Accordingly, they form the expectation that the one-object sample must be blue because it corresponds to the full population, which, again, infants have in front of them.

Data collected by Denison and Xu (2010) seem to support this interpretation. The authors showed 14-month-old infants one transparent jar containing 50 lollipops of two colors, with a ratio of 4:1, and another jar with the same number of lollipops but with the ratio between colors inverted. The authors first established what color each individual infant preferred. Then, they presented the two jars to infants and two empty cups. After infants saw the content of the jars, one lollipop from the first jar was put in one of the cups and a lollipop from the second jar was put in the second cup. The action was executed in such a way that infants could see from what jar the lollipops were drawn but not the lollipops' colors. Finally, infants were left free to grab the cup they preferred. Denison and Xu found that infants chose the cup containing the lollipop taken from the jar with the higher proportion of lollipop of their favorite colors. This result does not have a straightforward interpretation in terms of the theory we proposed. It is arguably implausible to suppose that infants solve the task by relating the two possible outcomes (a pink lollipop or a black lollipop) to the space of logically possible outcomes afforded by the objects present in the scene. Representing such logical space would involve representing a set of distinct, mutually exclusive, possible events that is just too big to be represented.

However, other results seem easier to account for in the framework we propose but do not have a straightforward explanation in terms of intuitive statistics, insofar as this explanation requires a computation based on a perceived population distribution. Téglás et al. (2007) and) tested infants and children with a second type of scene, which differed in one crucial aspect from the lottery scenes (Fig. 1.1, right). It presented a ball bouncing inside a box with one hole in a vertical wall and three holes in the opposite vertical wall. After some time in which infants and children could see the ball freely bouncing inside, an occluder covered the box. Care was taken to ensure that the position and last trajectory of the ball right before the occlusion was uninformative about the side from which it would eventually exit. Finally, the ball exited the box from one of the sides.) showed that 12-month-old infants looked longer at the improbable exit (the one-hole side) than at the probable exit. Likewise showed that children react faster when they have to guess that the ball will exit the three-hole side.

Just as the lottery scene, this kind of scene individuates four logically possible outcomes. The ball can exit from the only hole on the one side or else from the upper, the middle, or the lower holes on the other side. Thus, according to our theory of intuitive probability, reasoners should expect the ball to exit from the three-hole side, rather than from the one-hole side because one kind of events (say, exiting from the left side) occurs in three possible future continuations of the scene whereas another kind of events (say, exiting from the right side) only occurs in one possible future continuation. Indeed, infants and children behaved as predicted by our theory. However, is it possible to make the same prediction in terms of an intuitive statistics theory? We do not find it that simple. According to this theory, the ability to estimate the probability of a future single event's probability is derivative of the ability to grasp the relation between a sample and a perceived population. However, in the case of the ball in the box, there is no perceivable population distribution of which the exit of the ball from a particular side is a sample. Whatever happens, it happens only in the mind of the observer: if infants and children anticipate where the ball will exit, they do it by constructing a problem space of mental possibilities, on the basis of which they form their intuitions of the probability of future outcomes.

7. INFANT RATIONALITY AND SIMULATIONS: YET A THIRD ALTERNATIVE?

Recently Téglás et al. (2011) showed that infants' probabilistic reasoning abilities are even more sophisticated than what we have been discussing so far. As we recalled, any scene contains multiple kinds of information. An optimal reasoner should be able to decide which ones are relevant in a particular situation, as well as weight their relevance for the problem at hand. Furthermore, such evaluations must adapt dynamically because small changes in a developing situation can change their relative importance. Téglás et al. (2011) operationalized the investigation on infants' abilities to weight and integrate different changing cues in their predictions about future events by modifying crucial aspects of the scenes tested by Téglás et al. (2007). Infants were always presented with simple situations in which three yellow and one blue object bounced inside a lottery-like container. However, first, the objects of either categories could be either far or distant from the exit before the occluder covered the content of the box (Fig. 1.1). Second, the length of the occlusion separating the last moment in which the objects were visible before the occlusion and the first moment in which one of the object exited the container varied. Thus, the experiments varied the relevance of two cues: how distant an object is from the exit and how many objects of different classes are in the container.

The logic of these experiments was as follows. When objects cannot be seen for a very short time because an extremely short occlusion hides them from sight, then the distance of an object from the exit, and not its class membership, should be the most relevant cue to predict the next future outcome. Indeed, it would be almost impossible in an extremely short time for a distant object to exit the container; so, infants should expect that the first object to exit the container is the one that was closest to the exit before the short occlusion, regardless of its class membership. On the other hand, when the objects cannot be seen for a long period, because they keep moving inside the container, their locations prior to the occlusion is irrelevant. Surely, they changed positions many times during the occlusion period and hence where they were the last time one could see them is not predictive of where they will be after the occlusion. However, the shape and color of the object do not change. Thus, in this case, only the class membership of the objects (whether they belong to the most represented or to the less represented class) should matter. Thus, an optimal rational prediction of the outcomes should consider at least two aspects of the situation: the number of objects per category and the relative distance of the objects from the exit. Moreover, the relative importance of these two factors should vary depending on the length of the occlusion. Indeed, as indicated by their looking time at the final outcomes, infants only considered the relative distance of the objects from the exit, but not their categories or category distribution, when the occlusion was 0.04-s long. When the occlusion increased to 1 s, they considered both factors. Finally, when the occlusion was 2-s long, they only considered the category membership of the objects, but not their distance from the exit, to form their expectations. Thus, infants behaved as optimal rational agents. Téglás et al. (2011) also specified the formal way in which infants can be said to be optimal rational agents. They proposed a probabilistic model in which a Bayesian ideal observer equipped with basic knowledge of an objects' physical properties quantitatively predicts infants' looking behavior. Intuitively, the model simulates the possible trajectories of the objects as temporal series of possible world states, where the expectation of an outcome (i.e. one blue object exits first at a given time) is a function of how many trajectories are compatible with that outcome. The model fits impressively well with the infants' looking times in this as well as in many other studies, opening the possibility that a Bayesian inference system could be part of the explanation of many known results in infants' cognition of objects and events in the world.

Is the explanation proposed by Téglás et al. (2011) yet a third theory of infants' reasoning about the uncertain future? The fact that the model uses the simulation of physical trajectories to make its prediction may suggest that infants' expectations are based on simulation mechanisms reproducing object trajectories analogically, a view akin to a mental model theory of mental processes. Although this is a possibility, it is in no way mandated by the model proposed by Téglás et al. (2011). Such a model does not carry any commitment to the format of the internal representations on the basis of which infants form their expectations. That is, it is compatible both with a view of mental processes that takes seriously the existence of "analog simulations" in the mind or else with a view that conceives simulations as knowledge-based symbol manipulations. The case is no different from the debate about the nature of imagery (Pylyshyn, 1973, 1980, 2002, 2007).

Although the exact format of the internal representation underlying infants' abilities at integrating different information cannot be established, adult data suggest that humans are extremely poor at reconstructing in imagination quite simple dynamical scenes, giving chronometric and explicit responses that seem incompatible with the presence of an underlying analog simulation (Levillain & Bonatti, 2011). These data would seem to cast doubts about the existence of analog simulations in infants as well.

Our point, in this context, is that the Bayesian model elaborated by Téglás et al. (2011) does not force upon us a theory that assumes the existence of mental analog simulations. Indeed, the details of the simulations are not so important. Téglás et al. showed that the simulations needed to account for infants' behavior can be dramatically curtailed—from several thousands to four—without loss of predictability (Téglás et al., 2011, supplementary material). The resilience of the model under severe resource limitations suggests that the specific details of the simulations are not what that carries its explanatory power but, rather, the ability to represent alternative situations and conceive their possible outcomes. What, instead, cannot be eliminated or reduced from the model is the presence of a sophisticated inference device incorporating logical operations about real and possible future states (Perfors, Tenenbaum, Griffiths, & Xu, 2011). The postulation of the existence of this particular representational ability, which is highly consistent with the theory we proposed, is what we believe will become one of the major focuses of research in the next years.

8. WHAT ABOUT "EXPERIENCED FREQUENCIES"?

We have discussed two theories about intuitive understanding of probabilities, and we have tried to speculate about how they could account for some results about infants' probabilistic reasoning. We have shown that they can both predict some of the existing data but also that both have difficulties at predicting other data. The discussion may suggest that infants may have access to different computations that both allow, in certain cases, to predict the probability of single future events. Infants can predict a future event by seeing it as a sample of a perceived distribution. They can also predict a future event representing a space of logically possible outcomes and locating it inside that space. Further research is needed to explore this possibility. However, one point that the data and the explanations we presented should drive home is that infants' reasoning about the probable future cannot be explained on the basis of elementary perceptual biases nor on the basis of simple frequency mechanisms. To which we finally turn.

Both our proposal and the intuitive statistics theory postulate computational mechanisms that cannot be reduced to the ubiquitous frequency detection abilities that have been often documented in infants and adults. What, then, is the role of the "encountered frequency" of events that, according to the frequentist theory, are at the basis of our reasoning about distributions? We do not know whether and how infants integrate frequency information potentially in conflict with their initial intuitions of probability, but Téglás et al. explored this issue at least in 5-year olds. Children were shown situations where a ball inside a container with three exits on one side and one exit on the other side bounces randomly, eventually exiting the container (Fig. 1.1, right). The intuitive probability afforded by the device should lead one to predict a more likely exit from the three-exit side. However, by repeatedly presenting a scene ending with the ball exiting from the one-exit side, a conflict between a priori probability afforded by the situation and actual frequency of outcomes arose. Children had to press a button when the ball exited the container after an occlusion period that obliged them to react from their representations of the scene. Before their responses, they were also asked to give explicit judgments about what outcome they thought more likely. Finally, after being exposed for a while to the actual frequencies of outcomes, they were asked again an explicit judgment about what outcome occurred most.

Initially, both children's motor responses and their explicit judgments were influenced by the intuition that the ball had more chances to exit from the three-exit side. Also, after experiencing the frequencies of outcomes, when such frequency conflicted with the initial intuitions, children motor responses adapted to the distribution: after some time, children reacted faster when the ball exited the one-hole side. However, their explicit judgments did not adapt to frequencies. Even after seeing the ball exiting the one-exit side 75% of the time, children maintained that the ball was more likely to exit the three-hole side, as if the main factor determining their explicit judgments were, not their experience with the outcomes, but the representation of the logical possibilities afforded by the scene. So frequency does have an effect, but not where the frequentists would predict it. In our experiments, it affected motor responses, but not judgments about probabilities. That is, experience molds the way we act, but not necessarily the way we think.

9. THE FUTURE OF PREDICTIONS ABOUT THE FUTURE

The theory we proposed asks us to seriously consider the possibility that infants are little logicians, who can create logical representations of situations and make inferences from them in a rational way. Infants do seem to use abstract logical operators when acquiring rules (Marcus, Vijayan, Rao, & Vishton, 1999), creating representations of sets (Feigenson & Halberda, 2004), or learning words (Halberda, 2003). And by now there is good evidence that toddlers spontaneously engage in a form of exploratory play that can be described as a kind of rational hypothesis testing and confirmation (Bonawitz et al., 2010; Cook, Goodman, & Schulz, 2011; Gweon & Schulz, 2011; Schulz, Standing, & Bonawitz, 2008), suggesting the presence of very advanced logical representations needed to formulate and test hypotheses. But researchers have just begun to scratch the surface of the problem of determining the nature of infants' logical representations, and evidence to inform us about the existence, the format, and the extent of such representations is entirely lacking. Equally poor is our understanding of the relation between the representations of future possibilities afforded by a scene and infants' different systems for representing quantities. These are questions that we find fascinating and that will draw our attention in the coming years.

Yet, even in the current defective state of knowledge, we now know that infants can predict the uncertain future far better than a picture of early cognition as a collection of different encapsulated systems would suggest. This conclusion does not square with most theories about human reasoning and certainly seriously puts into question the currently popular dual process approach to human reasoning. There is an empirical and theoretical space that most of that literature missed. It demands to be explored and perhaps will help us tell a piece of the story about the inventiveness of human thought.

ACKNOWLEDGMENTS

The research was supported by the Ministerio de Ciencia e Innovación PSI2009-08232PSIC grant to LLB, by Consolider Ingenio 2010 Program (CSD2007-00012) of the Spanish Ministry of Economy and Competitiveness, by the Secretaria de Universitats i Recerca del Departament d'Economia i Coneixement of the Generalitat de Catalunya (2011 FIB_000695), and by the Hungarian Science Foundation (OTKA NK83997) grant. We thank Kimberly Brink, Fei Xu, Gergo Csibra, and Tamar Kushnir for their comments on a first version of the manuscript.

REFERENCES

Amati, D., & Shallice, T. (2007). On the emergence of modern humans. *Cognition, 103*(3), 358–385.

Baillargeon, R., Spelke, E. S., & Wasserman, S. (1985). Object permanence in five-month-old infants. *Cognition, 20*(3), 191–208.

Bonatti, L. (1994). Propositional reasoning by model? *Psychological Review, 101*(4), 725–733.

Bonawitz, E. B., Ferranti, D., Saxe, R., Gopnik, A., Meltzoff, A. N., Woodward, J., et al. (2010). Just do it? Investigating the gap between prediction and action in toddlers' causal inferences. *Cognition, 115*(1), 104–117.

Braine, M. D. S., O'Brien, D. P., Noveck, I. A., Samuels, M. C., Lea, R. B., Fisch, S. M., et al. (1995). Predicting intermediate and multiple conclusions in propositional logic inference problems: further evidence for a mental logic. *Journal of Experimental Psychology: General, 124*(3), 263–292.

Carruthers, P. (2002). The cognitive functions of language. *Behavioral and Brain Sciences, 25*(6), 657–726.

Cook, C., Goodman, N. D., & Schulz, L. E. (2011). Where science starts: spontaneous experiments in preschoolers' exploratory play. *Cognition, 120*(3), 341–349.

Cosmides, L. (1989). The logic of social exchange: has natural selection shaped how humans reason? Studies with the Wason selection task. *Cognition, 31*(3), 187–276.

Cosmides, L., & Tooby, J. (1996). Are humans good intuitive statisticians after all? Rethinking some conclusions from the literature on judgment under uncertainty. *Cognition, 58*(1), 1–73.

De Neys, W. (2012). Bias and conflict: a case for logical intuitions. *Perspectives on Psychological Science,* 7(1), 28–38.

De Neys, W., & Franssens, S. (2009). Belief inhibition during thinking: not always winning but at least taking part. *Cognition, 113*(1), 45–61.

De Neys, W., & Schaeken, W. (2007). When people are more logical under cognitive load: dual task impact on scalar implicature. *Experimental Psychology, 54*(2), 128–133.

Denison, S., & Xu, F. (2010). Twelve- to 14-month-old infants can predict single-event probability with large set sizes. *Developmental Science, 13*(5), 798–803.

Evans, J. St. B. T. (1989). *Bias in human reasoning: Causes and consequences*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Evans, J. St. B. T. (2003). In two minds: dual-process accounts of reasoning. *Trends in Cognitive Sciences,* 7(10), 454–459.

Evans, J. St. B. T. (2008). Dual-processing accounts of reasoning, judgment, and social cognition. *Annual Review of Psychology, 59*, 255–278.

Feigenson, L. (2005). A double-dissociation in infants' representations of object arrays. *Cognition, 95*(3), B37–b48.

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object arrays. *Cognition, 97*(3), 295–313.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314.

Feigenson, L., & Halberda, J. (2004). Infants chunk object arrays into sets of individuals. *Cognition, 91*(2), 173–190.

Fiser, J., & Aslin, R. N. (2002). Statistical learning of new visual feature combinations by infants. *Proceedings of the National Academy of Sciences of the United States of America, 99*(24), 15822–15826.

Gergely, G., & Csibra, G. (2003). Teleological reasoning in infancy: the naïve theory of rational action. *Trends in Cognitive Sciences,* 7(7), 287–292.

Gergely, G., Nádasdy, Z., Csibra, G., & Bíró, S. (1995). Taking the intentional stance at 12 months of age. *Cognition, 56*(2), 165–193.

Gigerenzer, G., & Hoffrage, U. (1995). How to improve Bayesian reasoning without instruction: frequency formats. *Psychological Review, 102*(4), 684–704.
Gilovich, T., Vallone, R., & Tversky, A. (1985). The hot hand in basketball: on the misperception of random sequences. *Cognitive Psychology, 17*(3), 295–314.
Griffiths, T. L., Kemp, C., & Tenenbaum, J. B. (2008). Bayesian models of cognition. In R. Sun (Ed.), *The Cambridge Handbook of Computational Psychology* (pp. 59–100). Cambridge: Cambridge University Press.
Griffiths, T. L., & Tenenbaum, J. B. (2006). Optimal Predictions in Everyday Cognition. *Psychological Science, 17*(9), 767–773.
Gweon, H., & Schulz, L. (2011). 16-Month-olds rationally infer causes of failed actions. *Science, 332*(6037), 1524.
Halberda, J. (2003). The development of a word-learning strategy. *Cognition, 87*(1), B23–B34.
Hauser, M. D., Newport, E. L., & Aslin, R. N. (2001). Segmentation of the speech stream in a non-human primate: statistical learning in cotton-top tamarins. *Cognition, 78*(3), B53–B64.
Hume, D. (2000). *An enquiry concerning human understanding*. Kitchener, ON: Batoche.
Johnson-Laird, P. N., Legrenzi, P., Girotto, V., Legrenzi, M. S., & Caverni, J.-P. (1999). Naive probability: a mental model theory of extensional reasoning. *Psychological Review, 106*(1), 62–88.
Johnson-Laird, P. N. (1983). *Mental models*. Cambridge, MA: Harvard University Press.
Johnson-Laird, P. N., Byrne, R. M., & Schaeken, W. (1992). Propositional reasoning by model. *Psychological Review, 99*(3), 418–439.
Kahneman, D., & Tversky, A. (1972). Subjective probability: a judgment of representativeness. *Cognitive Psychology, 3*(3), 430–454.
Kushnir, T., Xu, F., & Wellman, H. M. (2010). Young children use statistical sampling to infer the preferences of other people. *Psychological Science, 21*(8), 1134–1140.
Lea, R. B., & Mulligan, E. J. (2002). The effect of negation on deductive inferences. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 28*(2), 303–317.
Lea, R. B., O'Brien, D. P., Fisch, S. M., & Noveck, I. A. (1990). Predicting propositional logic inferences in text comprehension. *Journal of Memory and Language, 29*(3), 361–387.
Levillain, F., & Bonatti, L. L. (2011). A dissociation between judged causality and imagined locations in simple dynamic scenes. *Psychological Science, 22*(5), 674–681.
Marcus, G. F., Vijayan, S., Rao, S. B., & Vishton, P. M. (1999). Rule learning by seven-month-old infants. *Science, 283*(5398), 77–80.
Mithen, S. J. (1996). *The prehistory of the mind: a search for the origins of art, religion and science*. London: Thames and Hudson.
O'Brien, D. P., Braine, M. D. S., & Yang, Y. (1994). Propositional reasoning by mental models? Simple to refute in principle and in practice. *Psychological Review, 101*(4), 711–724.
Perfors, A., Tenenbaum, J. B., Griffiths, T. L., & Xu, F. (2011). A tutorial introduction to Bayesian models of cognitive development. *Cognition, 120*, 302–321.
Piaget, J., Chomsky, N., & Piattelli-Palmarini, M. (1980). *Language and learning: the debate between Jean Piaget and Noam Chomsky*. Cambridge, MA: Harvard University Press.
Pylyshyn, Z. W. (1973). What the mind's eye tells the mind's brain: a critique of mental imagery. *Psychological Bulletin, 80*(1), 1–24.
Pylyshyn, Z. W. (1980). Computation and cognition: issues in the foundations of cognitive science. *Behavioral and Brain Sciences, 3*(1), 111–169.
Pylyshyn, Z. W. (2002). Mental imagery: in search of a theory. *Behavioral and Brain Sciences, 25*(2), 157–238.
Pylyshyn, Z. W. (2007). *Things and places: How the mind connects with the world*. Cambridge, MA: MIT Press.
Reverberi, C., Bonatti, L. L., Frackowiak, R. S., Paulesu, E., Cherubini, P., & Macaluso, E. (2012). Large scale brain activations predict reasoning profiles. *Neuroimage, 59*(2), 1752–1764.

Reverberi, C., Pischedda, D., Burigo, M., & Cherubini, P. (2012). Deduction without awareness. *Acta Psychologica, 139*(1), 244–253.

Reverberi, C., Shallice, T., D'Agostini, S., Skrap, M., & Bonatti, L. L. (2009). Cortical bases of elementary deductive reasoning: inference, memory, and metadeduction. *Neuropsychologia, 47*(4), 1107–1116.

Saffran, J. R., Aslin, R. N., & Newport, E. L. (1996). Statistical learning by 8-month-old infants. *Science, 274*(5294), 1926–1928.

Schulz, L. E., Standing, H. R., & Bonawitz, E. B. (2008). Word, thought, and deed: the role of object categories in children's inductive inferences and exploratory play. *Developmental Psychology, 44*(5), 1266–1276.

Sloman, S. A. (1996). The empirical case for two systems of reasoning. *Psychological Bulletin, 119*(1), 3–22.

Southgate, V., & Csibra, G. (2009). Inferring the outcome of an ongoing novel action at 13 months. *Developmental Psychology, 45*(6), 1794–1798.

Spelke, E. S. (2003). What makes us smart? Core knowledge and natural language. In D. Gentner, & S. Goldin-Meadow (Eds.), *Language in mind: Advances in the study of language and thought* (pp. 277–311). Cambridge, MA: MIT Press.

Spelke, E. S., Breinlinger, K., Macomber, J., & Jacobson, K. (1992). Origins of knowledge. *Psychological Review, 99*(4), 605–632.

Stanovich, K. E., & West, R. F. (2000). Individual differences in reasoning: implications for the rationality debate? *Behavioral and Brain Sciences, 23*(5), 645–726.

Téglás, E., & Bonatti, L. L. (2008). Probability triggers the eye: reasoning about uncertain events in 12-month-old infants. *In: Proceedings from International Conference of Infant Studies*. Vancouver, CA.

Téglás, E., Girotto, V., Gonzalez, M., & Bonatti, L. L. (2007). Intuitions of probabilities shape expectations about the future at 12 months and beyond. *Proceedings of the National Academy of Sciences of the United States of America, 104*(48), 19156–19159.

Téglás, E., Vul, E., Girotto, V., Gonzalez, M., Tenenbaum, J. B., & Bonatti, L. L. (2011). Pure reasoning in 12-month-old infants as probabilistic inference. *Science, 332*(6033), 1054–1058.

Tenenbaum, J. B., Griffiths, T. L., & Kemp, C. (2006). Theory-based Bayesian models of inductive learning and reasoning. *Trends in Cognitive Sciences, 10*(7), 309–318.

Toro, J. M., & Trobalón, J. B. (2005). Statistical computations over a speech stream in a rodent. *Perception & Psychophysics, 67*(5), 867–875.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: heuristics and biases. *Science, 185*(4157), 1124–1131.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science, 211*(4481), 453–458.

Tversky, A., & Kahneman, D. (1993). Belief in the law of small numbers. In C. Keren, & C. Lewis (Eds.), *A handbook for riala attalysis ill the behavioral sciellces: Methodological issues* (pp. 341–349). Hillsdale, NJ: Erlbaum.

Wason, P. C. (1968). Reasoning about a rule. *Quarterly Journal of Experimental Psychology A, 20*(3), 273–281.

Woodward, A. L. (1998). Infants selectively encode the goal object of an actor's reach. *Cognition, 69*(1), 1–34.

Woodward, A. L. (1999). Infants' ability to distinguish between purposeful and non-purposeful behaviors. *Infant Behavior & Development, 22*(2), 145–160.

Xu, F., & Denison, S. (2009). Statistical inference and sensitivity to sampling in 11-month-old infants. *Cognition, 112*(1), 97–104.

Xu, F., & Garcia, V. (2008). Intuitive statistics by 8-month-old infants. *Proceedings of the National Academy of Sciences of the United States of America, 105*(13), 5012–5015.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*(1), B1–B11.